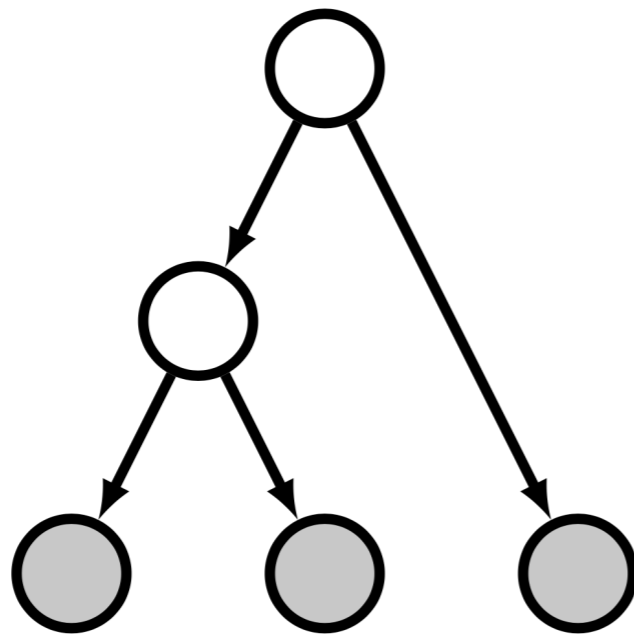


Intro. to Graphical Models and RevBayes

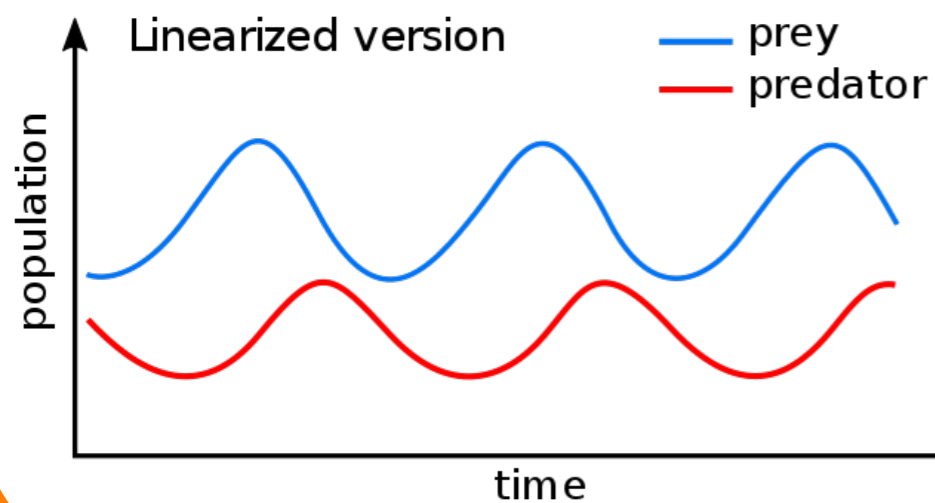


Jeremy M. Brown
Louisiana State University

What is a **model**?

Math

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$



Biology

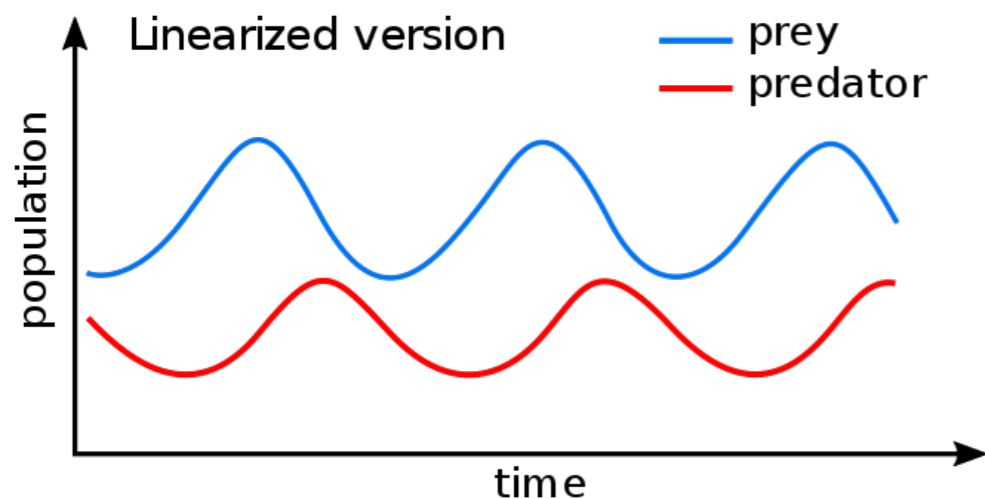
(something observed)



What is a **model**?

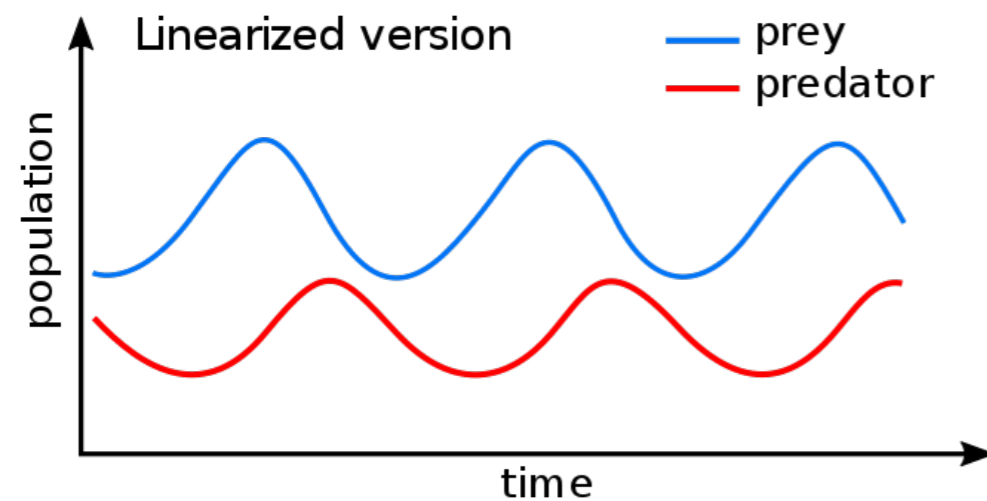
Mathematical models try to explain important features of (biological) systems using (relatively simple) mathematical principles.

$$\begin{aligned}\frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y,\end{aligned}$$



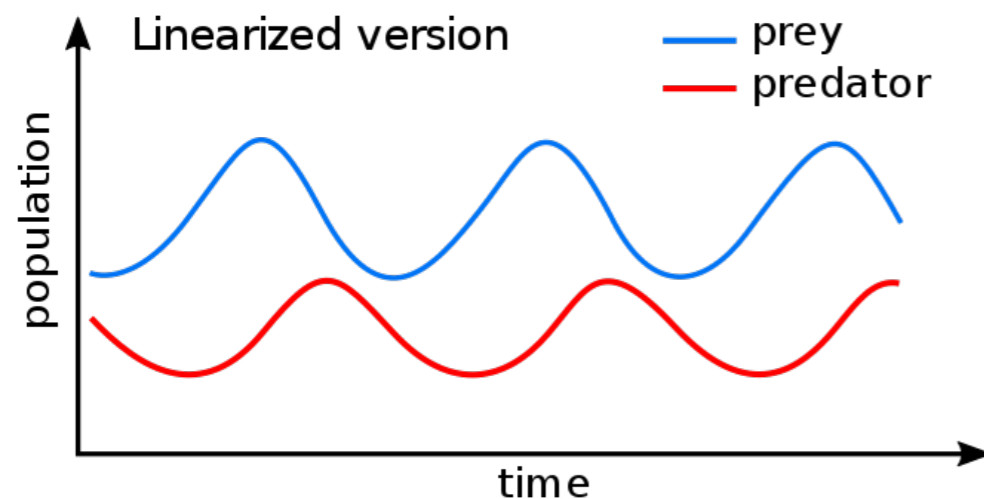
Deterministic and Stochastic Models

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$



Deterministic and Stochastic Models

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$



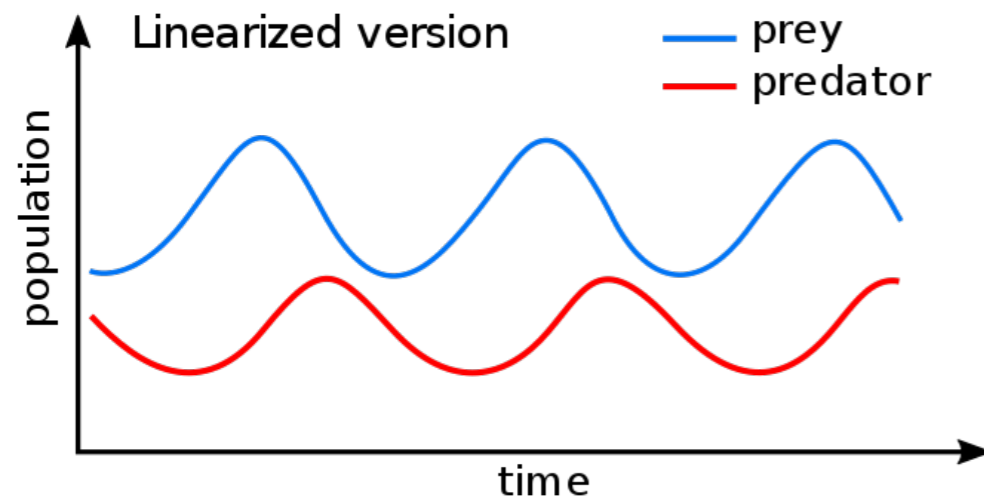
NO randomness

(no stochasticity)

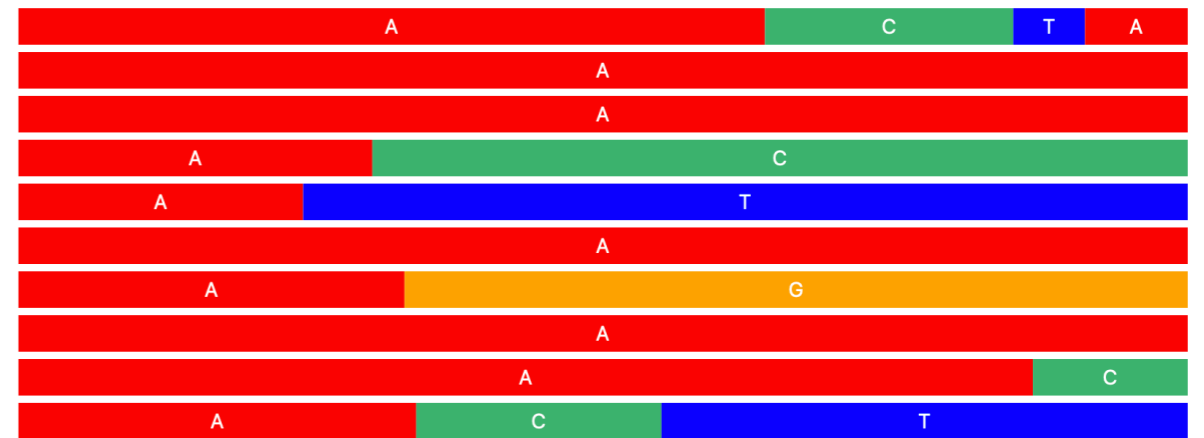
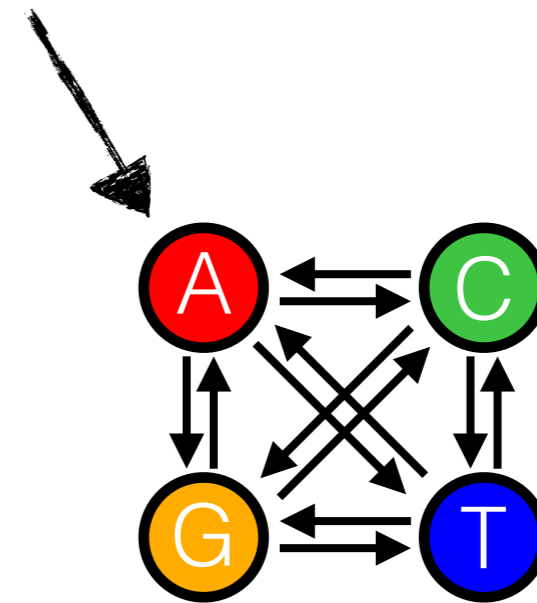
(no probability statements)

Deterministic and Stochastic Models

$$\begin{aligned}\frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y,\end{aligned}$$

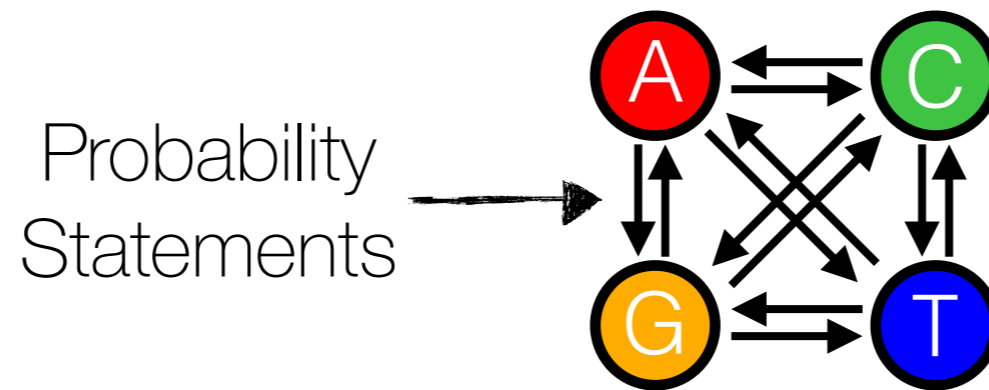


NO randomness
(no stochasticity)
(no probability statements)

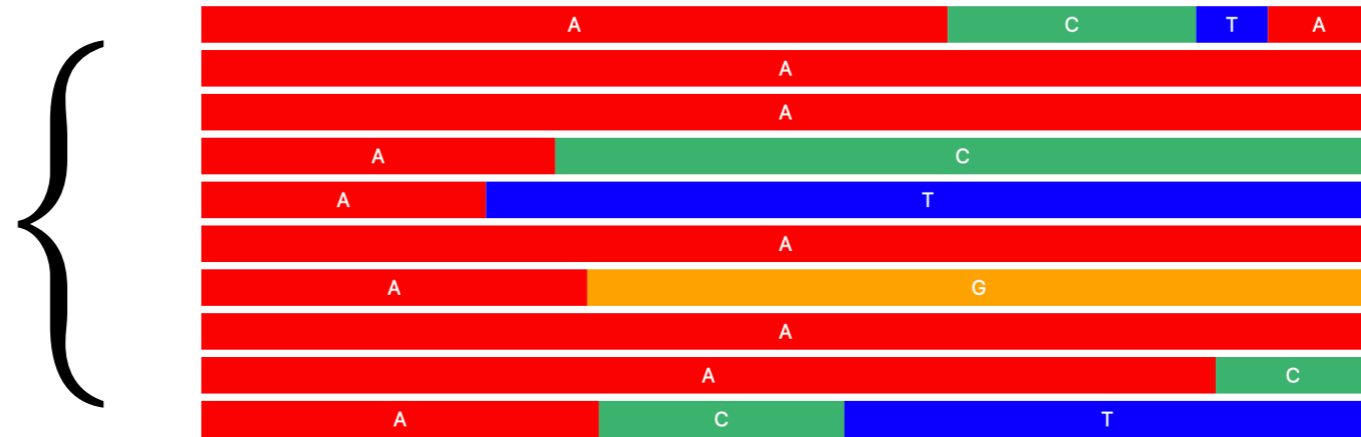


Randomness!
(stochasticity)
(probability statements)
(variation in possible outcomes)

Stochastic Models



Variable Outcomes



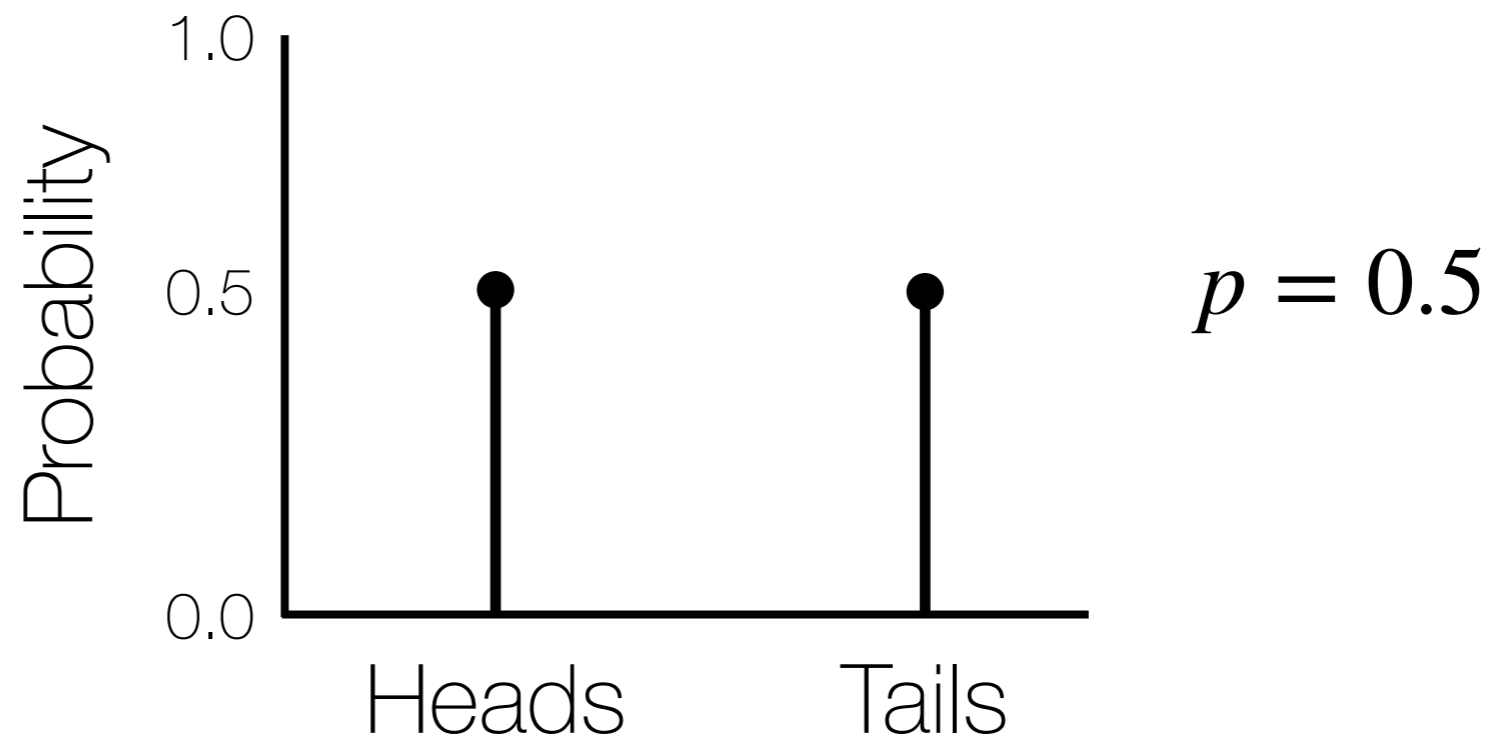
Randomness!

(stochasticity)

(probability statements)

(variation in possible outcomes)

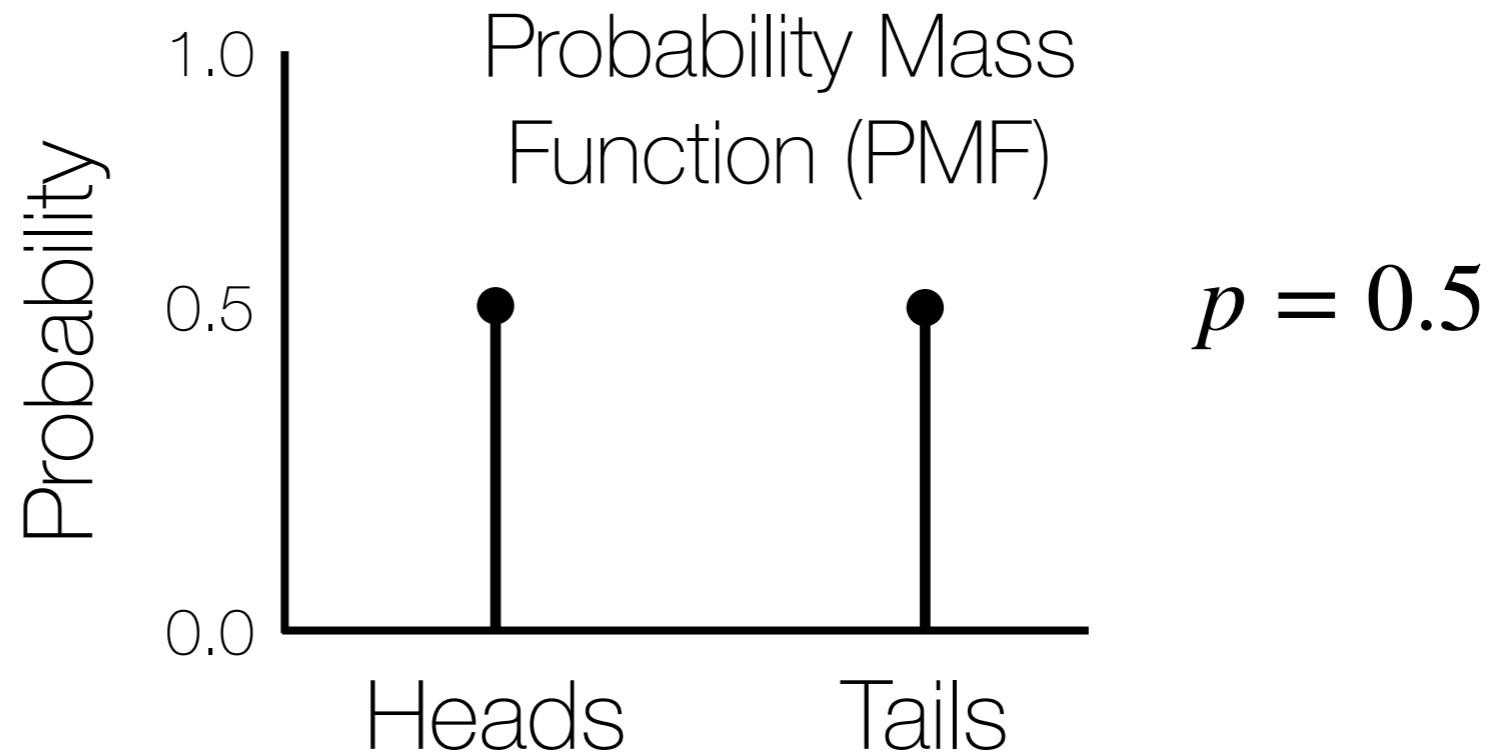
A simple **stochastic model**



Bernoulli

Describes the outcome of a single coin flip
(or other binary 0/1 trial)

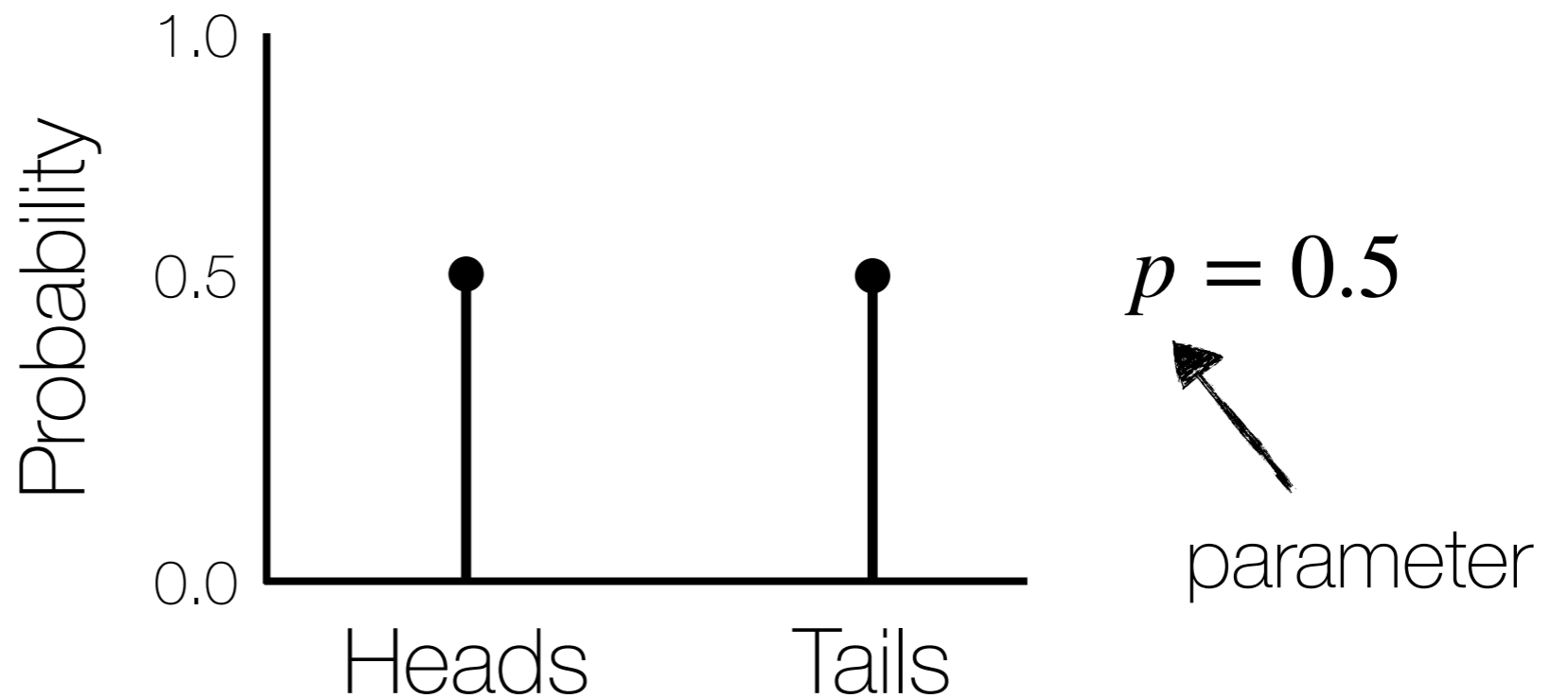
A simple **stochastic model**



Bernoulli

One type of *discrete* distribution

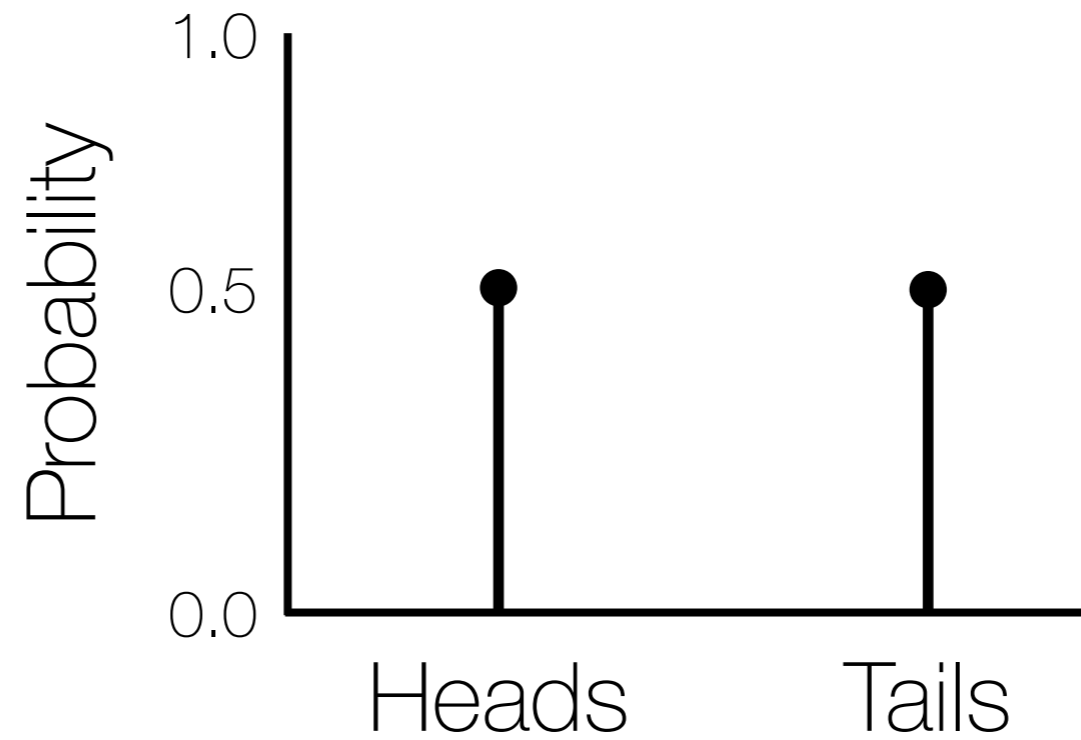
A simple **stochastic model**



$$P(\text{Heads}) = p$$

$$P(\text{Tails}) = 1 - p$$

Let's build this model!



$$p = 0.5$$

RevBayes



```
moleuser@amphioxys-49-of-66:~$ rb
```

```
RevBayes version (1.1.1)
```

```
Build from tags/1.1.1 (rapture-588-gae00cc) on Sat Mar 19 00:48:15 UTC 2022
```

```
Visit the website www.RevBayes.com for more information about RevBayes.
```

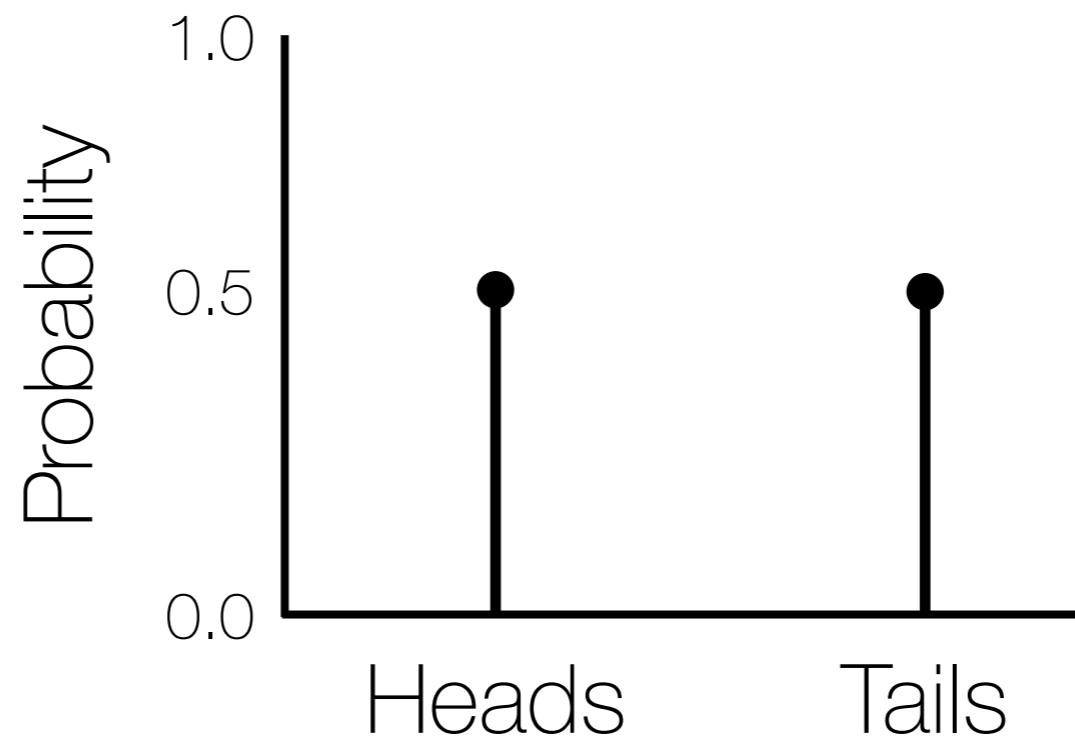
```
RevBayes is free software released under the GPL license, version 3. Type 'license()' for details.
```

```
To quit RevBayes type 'quit()' or 'q()'.
```

```
> _
```

<https://revbayes.github.io>

Let's build this stochastic model!

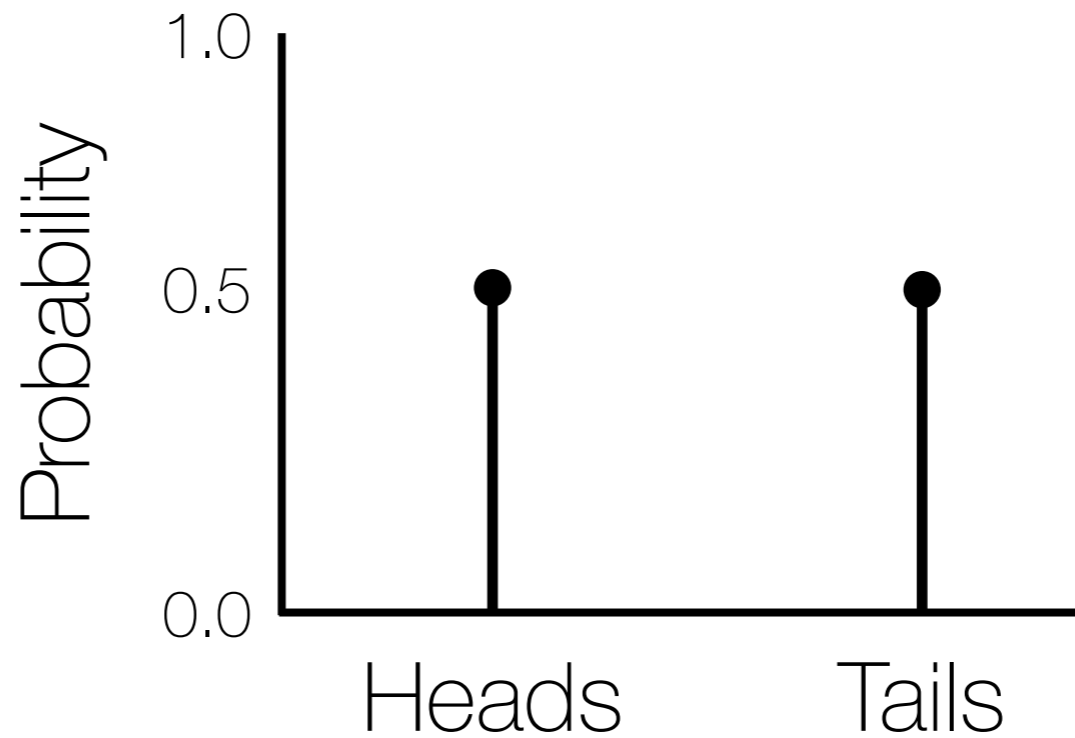


$$p = 0.5$$

$$z \sim \text{dnBernoulli}(0.5)$$

z

Let's build this stochastic model!



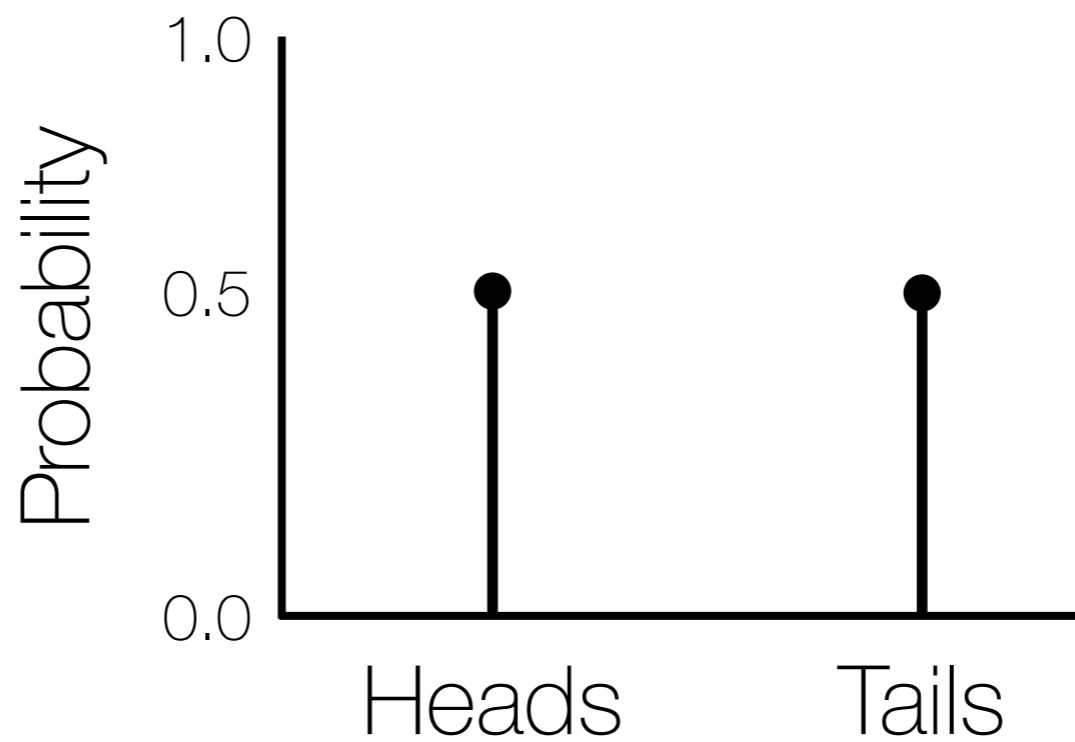
$$p = 0.5$$

Random Variable



“distributed as”

Rev language intended to feel a lot like R, but you can't create random variables in R.



```
> z ~ dnBernoulli(0.5)
[> z
  0
> z ~ dnBernoulli(0.5)
[> z
  0
> z ~ dnBernoulli(0.5)
[> z
  1
> z ~ dnBernoulli(0.5)
[> z
  1
> z ~ dnBernoulli(0.5)
[> z
  1
> z ~ dnBernoulli(0.5)
[> z
  1
> z ~ dnBernoulli(0.5)
[> z
  0
```

When you create a random variable in RevBayes, it automatically initializes that variable by drawing a value from the corresponding probability distribution.

The Role of Models

(1) Can we explain the **important features** of the real world using relatively **simple mathematical principles** (prediction)?

(2) Within this mathematical framework, can we **learn about the processes** that generated our data (inference)?

The most useful models can answer ‘yes’ to both.

$$z \sim \text{dnBernoulli}(0.5)$$

(1) 

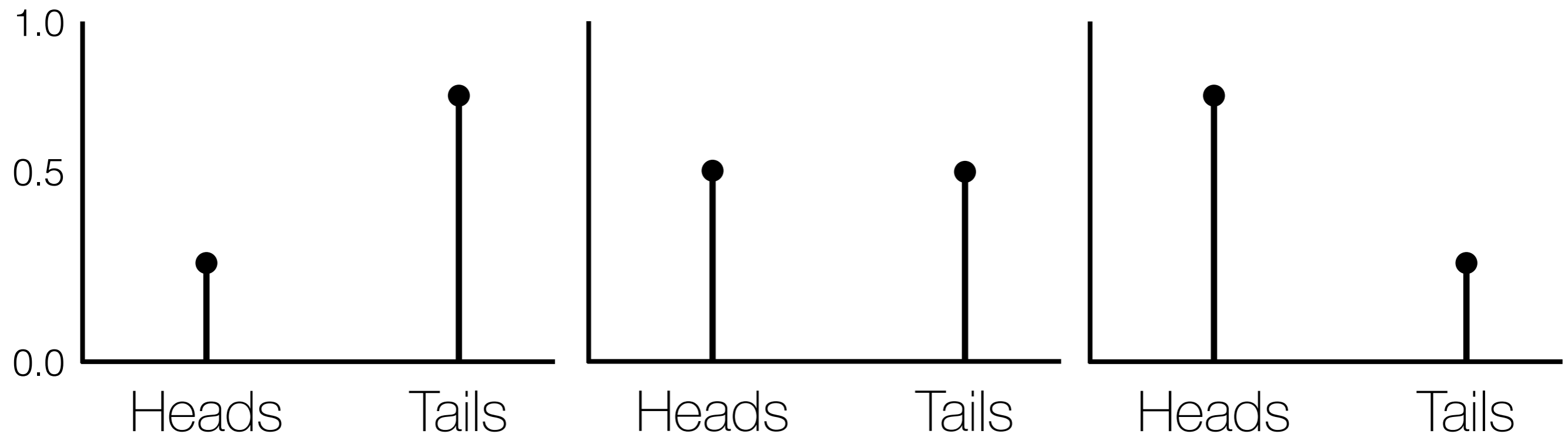
(2)  $p=0.5$
No data

Different parameter values imply different probabilities for outcomes
(different PMFs)

$$p = 0.25$$

$$p = 0.5$$

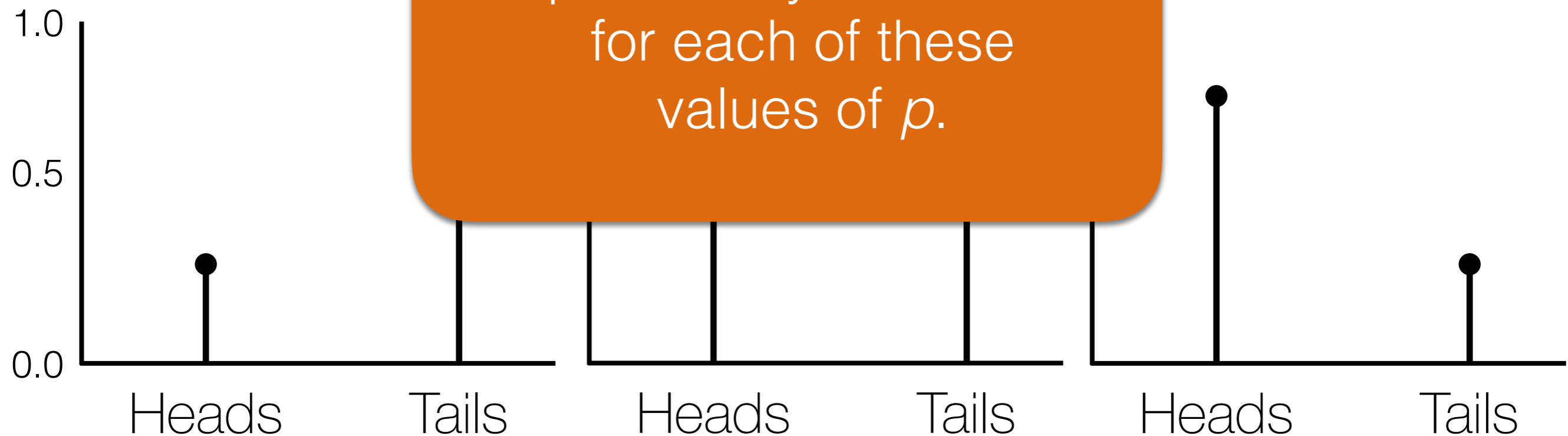
$$p = 0.75$$



Different parameter values imply different probabilities for outcomes (different PMFs)

$p = 0.25$

$p = 0.75$



Computing Likelihoods

```
p <- 0.5  
z ~ dnBernoulli(p)  
z.clamp(1)  
z.probability()
```

Here is where we attach our data (i.e., Heads) to our model. This is known as “clamping”.

Here we compute:

$$P(\text{Heads} \mid p = 0.5)$$

$$L(p = 0.5; \text{Heads})$$

```
> p <- 0.25
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
0.25
```

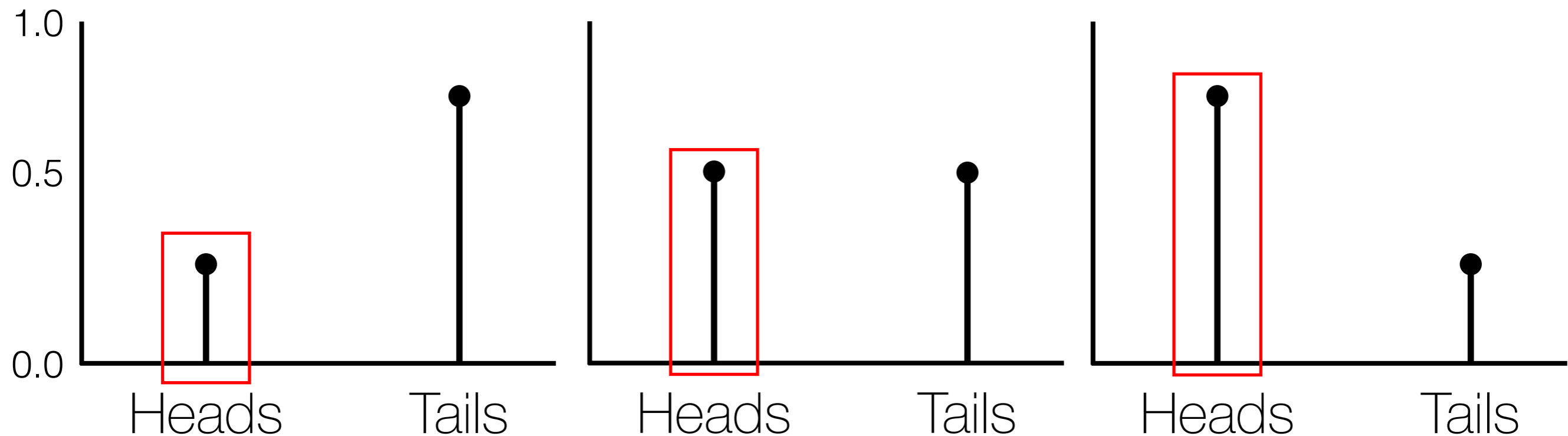
```
> p <- 0.5
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
0.5
```

```
> p <- 0.75
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
0.75
```

$p = 0.25$

$p = 0.5$

$p = 0.75$



```
> p <- 0.25
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
0.25
```

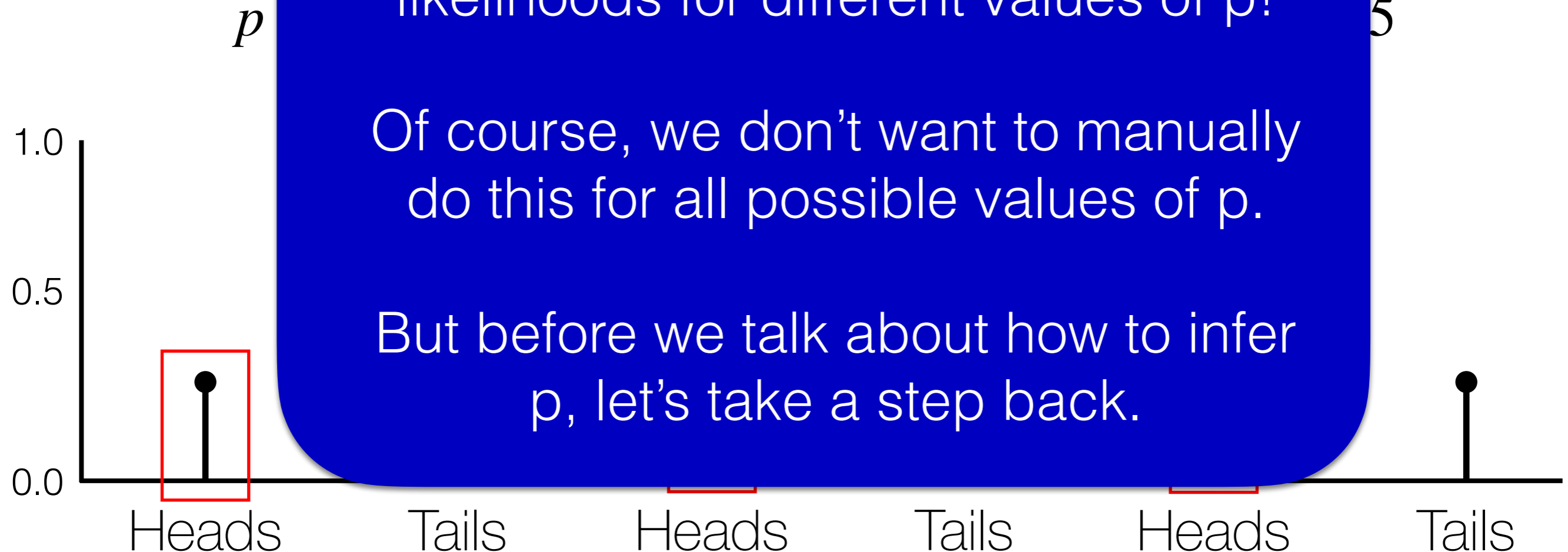
```
> p <- 0.5
> z ~ dnBernoulli(p)
> z.clamp(1)
```

```
> p <- 0.75
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
0.75
```

Being able to attach data to the model allows us to compare the likelihoods for different values of p !

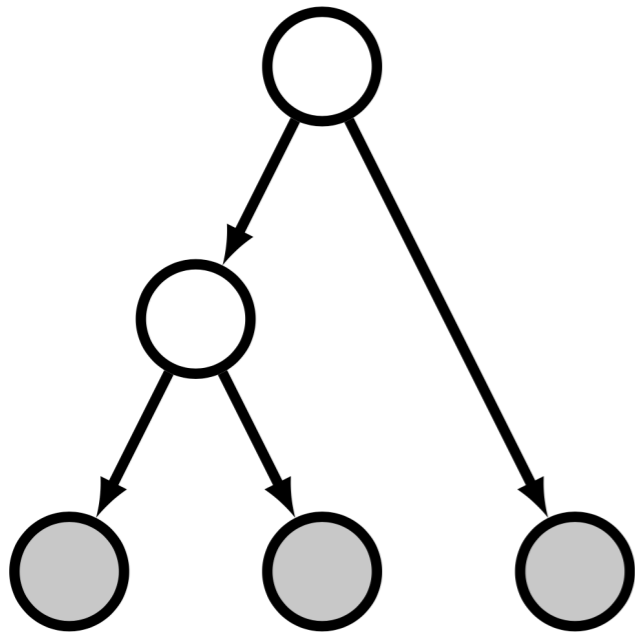
Of course, we don't want to manually do this for all possible values of p .

But before we talk about how to infer p , let's take a step back.



Graphical Models

Graphical models provide a means of depicting the dependencies among parameters in probabilistic models.

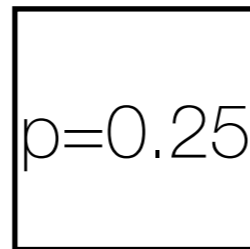


The “graphical” part of the name graphical models has to do with their basis on graphs, and not anything to do with drawing.

But it is convenient that graphical models can also be drawn, to clearly depict the structure of the model.

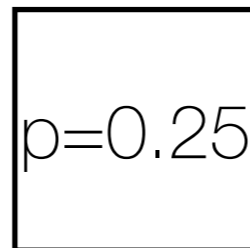
Building a Graphical Model

p is fixed at 0.25

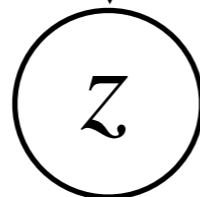


```
> p <- 0.25
```

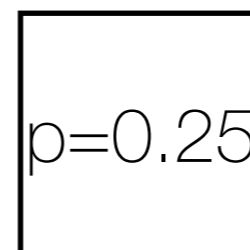
z depends on p
 z is a Bernoulli r.v.



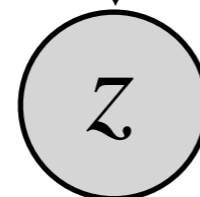
```
> z ~ dnBernoulli(p)
```

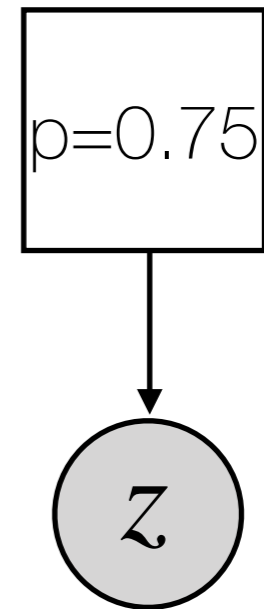
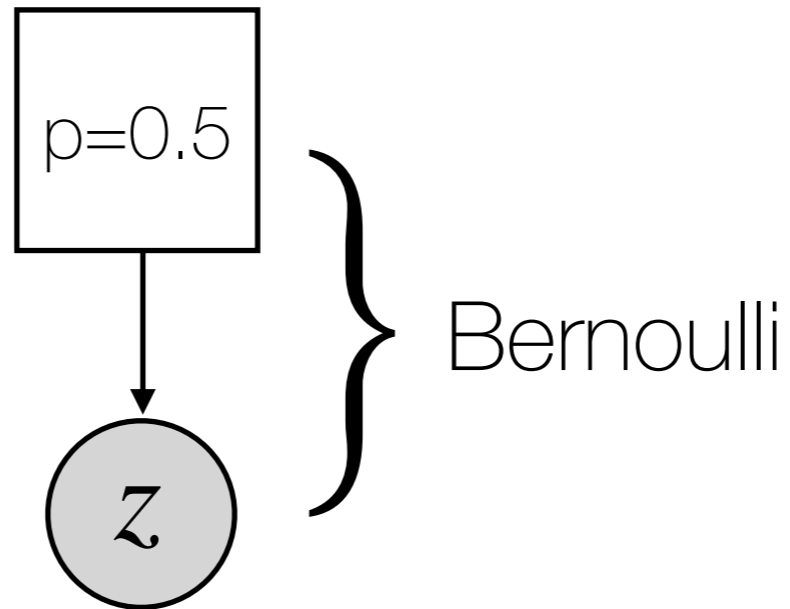
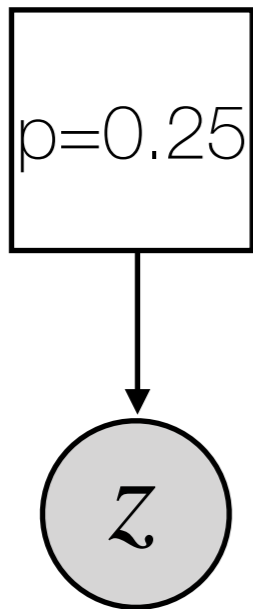


We “observed” a
value of 1 for z .



```
> z.clamp(1)
```





```
> p <- 0.25
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
  0.25
```

```
> p <- 0.5
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
  0.5
```

```
> p <- 0.75
> z ~ dnBernoulli(p)
> z.clamp(1)
> z.probability()
  0.75
```


What are the possible building blocks for graphical models?



RevBayes Syntax

```
sqrt(16)  
abs(-11)
```

**functions
and
arguments**

```
choose(6,3)  
ls()  
max([2.1,5.5,7.8])
```

```
test <- v(5,9,12)  
test  
test[2]
```

vectors

```
test[1] <- 5  
test[2] <- 9  
test[3] <- 12  
test
```

```
for (i in 1:5){  
  print(i)  
}
```

for loops

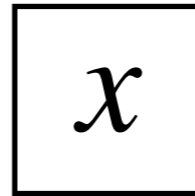
```
for (i in 1:3){  
  print(test[i])  
}
```

<https://revbayes.github.io/tutorials/intro/rev.html>

<https://revbayes.github.io/documentation/>

Graphical Model Syntax

Symbol



Type

Constant Node

Description

Constant nodes are like standard variables in a programming language. They are assigned fixed values.

Rev Example

```
x <- 2.3
```

Graphical Model Syntax

Symbol



Type

Deterministic Node

Description

Deterministic nodes depend on the values of other nodes, but in a deterministic (fixed) way. They have no probability distribution intrinsically associated with them.

Rev Example

$$y := 2 * x$$

Graphical Model Syntax

Symbol



Type

Stochastic Node

Description

Stochastic nodes represent random variables and take on values according to some probability distribution. These distributions may have parameters defined in other nodes in the model graph. The type of distribution associated with a node is often not written explicitly when a model is drawn, but it must be specified.

Rev Example

```
z ~ dnBernoulli(0.5)
```

Graphical Model Syntax

Symbol



Type

Clamped Stochastic Node

Description

Data can be viewed as the observed outcome of a stochastic node. Clamping involves assigning a set of observations to an associated stochastic node. Clamping data allows the values of other parameters in the model to be inferred.

Rev Example

```
z ~ dnBernoulli(0.5)
z.clamp(1)
```

Graphical Model Syntax

Symbol



Type

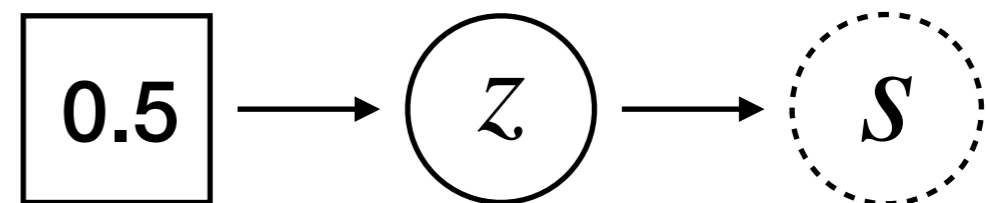
Conditional Dependency

Description

Arrows indicate conditional dependencies and are directional. The node (variable) that the arrow points to depends on the value of the node at the other end of the arrow. For instance, if p points to Z , then we say that Z is dependent on p .

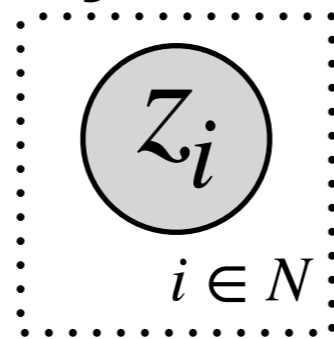
Rev Example

```
p <- 0.5
z ~ dnBernoulli(p)
s := z*3
```



Graphical Model Syntax

Symbol



Type

Plate

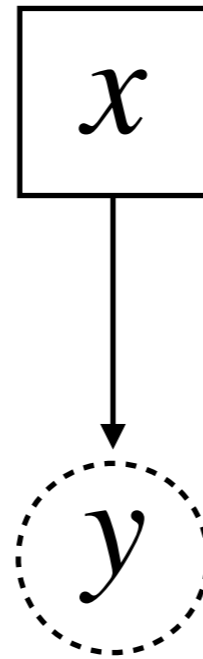
Description

Plates simply represent repetition and make it easier to depict graphical models with a repetitive structure. For instance, the example above shows N instances of the clamped variable, z , with each instance indexed by i . Plates can be used with any type of node, but are commonly used with clamped nodes representing data.

Rev Example

```
N <- 10
for (i in 1:N){
  z[i] ~ dnBernoulli(0.5)
  z[i].clamp(1)
}
```


Super Simple Models



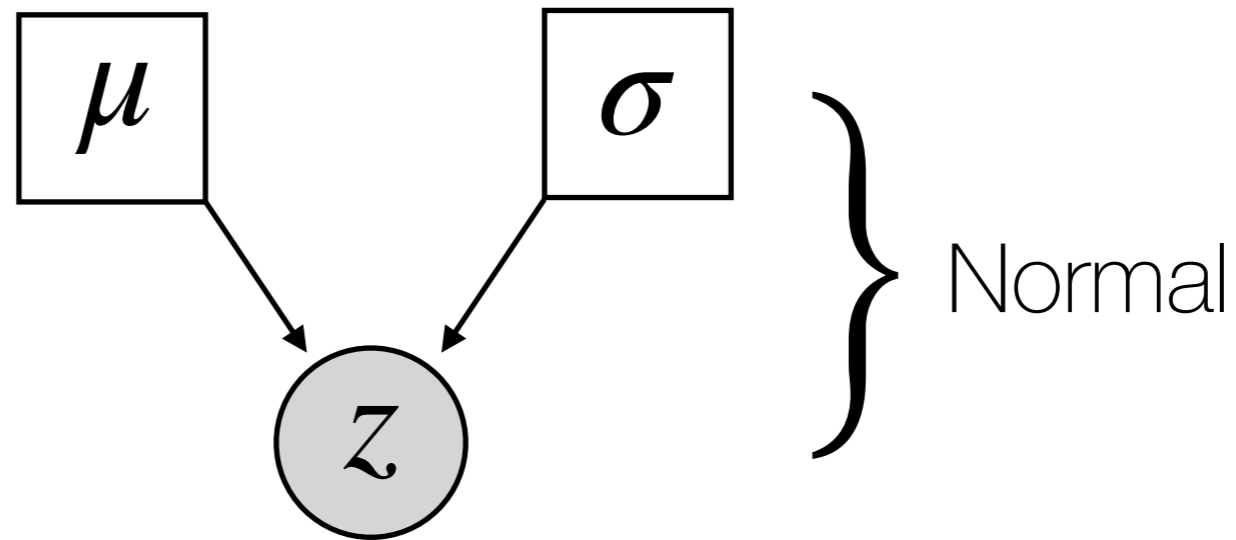
```
x <- 2  
y := x / 4
```

What's the structure of this model?

What would happen if x was set to 16?

Try changing x without changing y , but then look at value of y .

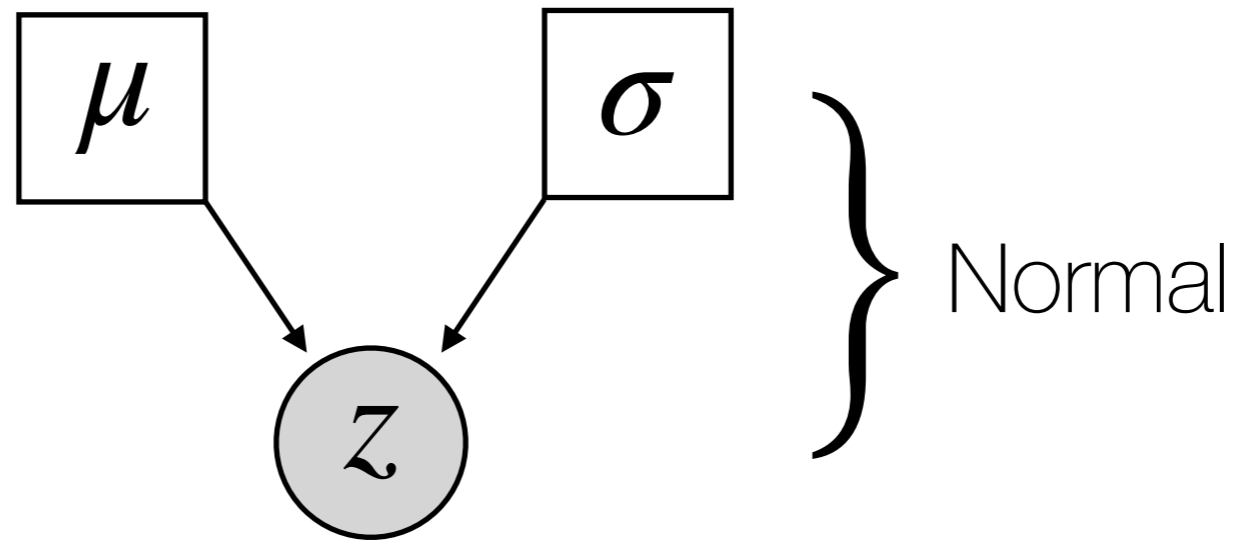
Super Simple Models



```
m <- 10  
s <- 0.1  
z ~ dnNormal(m,s)  
z.clamp(10.01)
```

What's the structure of this model?
What can we infer about this model?

Super Simple Models

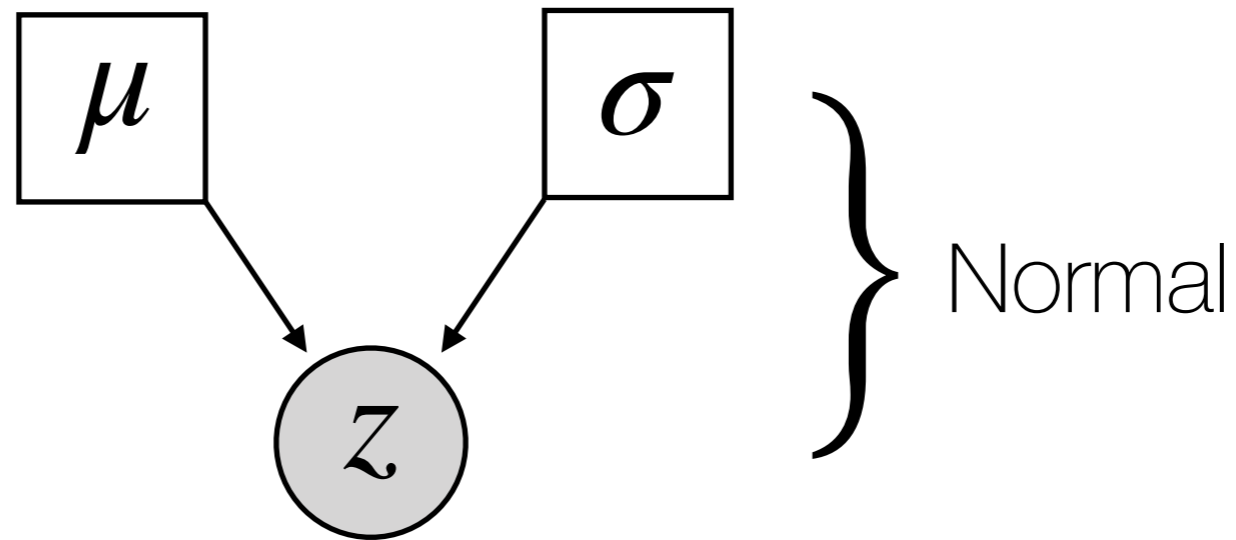


```
m <- 10  
s <- 0.1  
z ~ dnNormal(m,s)  
z.clamp(10.01)
```

What's the structure of this model?
What can we infer about this model?

Nothing to infer! Only nodes are constant or clamped.

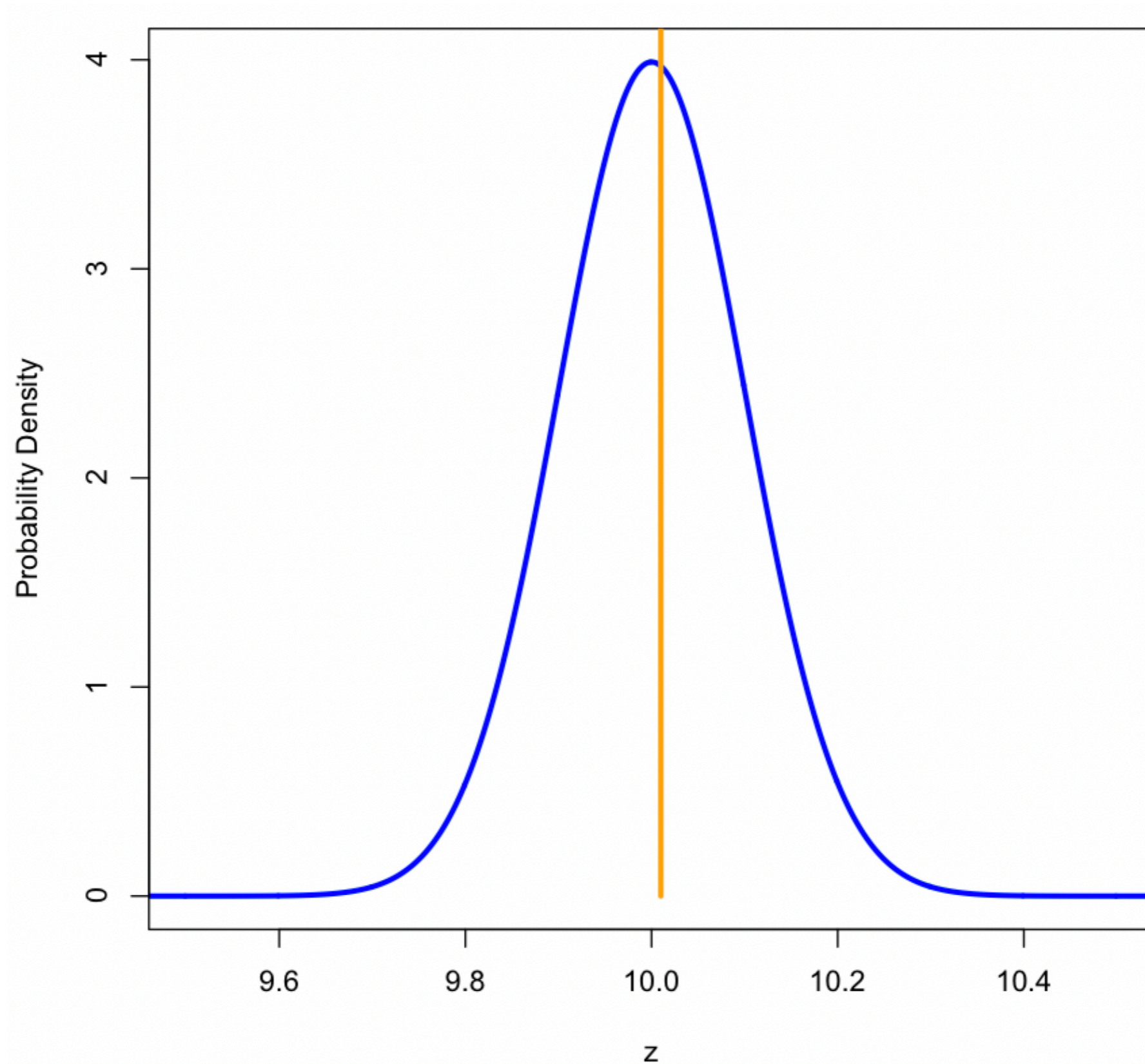
Super Simple Models



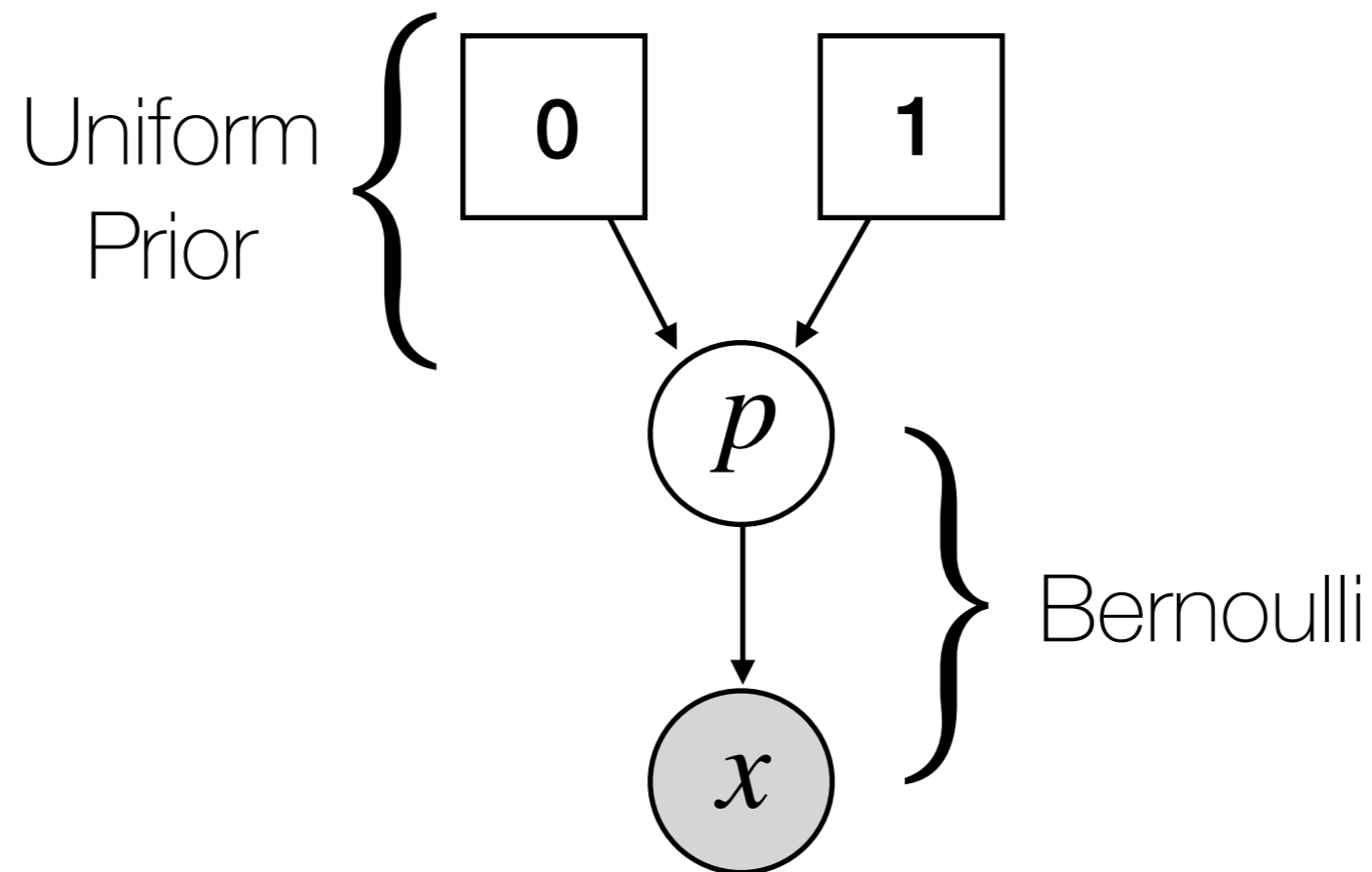
```
m <- 10  
s <- 0.1  
z ~ dnNormal(m,s)  
z.clamp(10.01)
```

What's the probability of our "observed" value?

Probability Density Function (PDF)

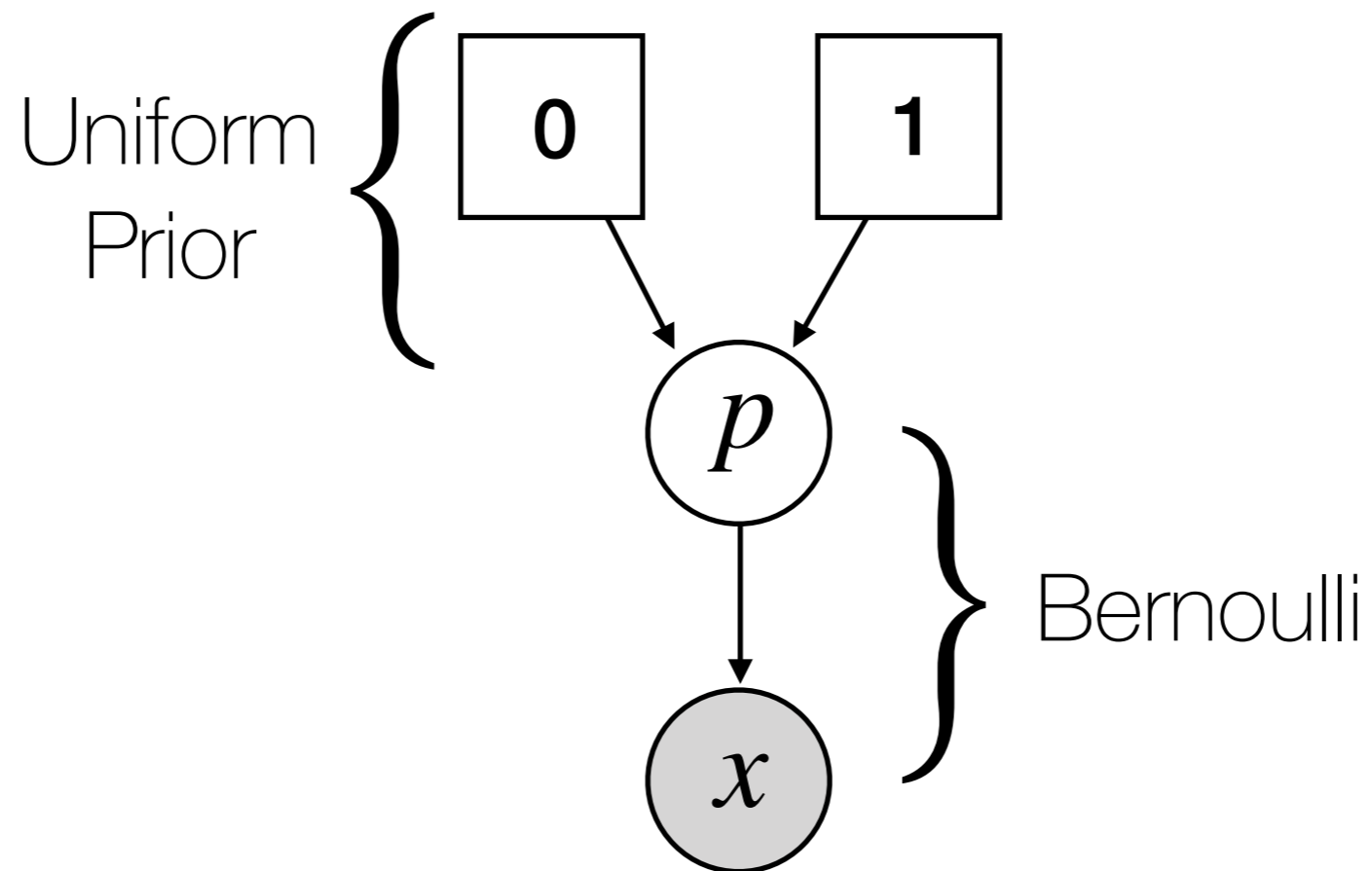


Single Bernoulli Trial



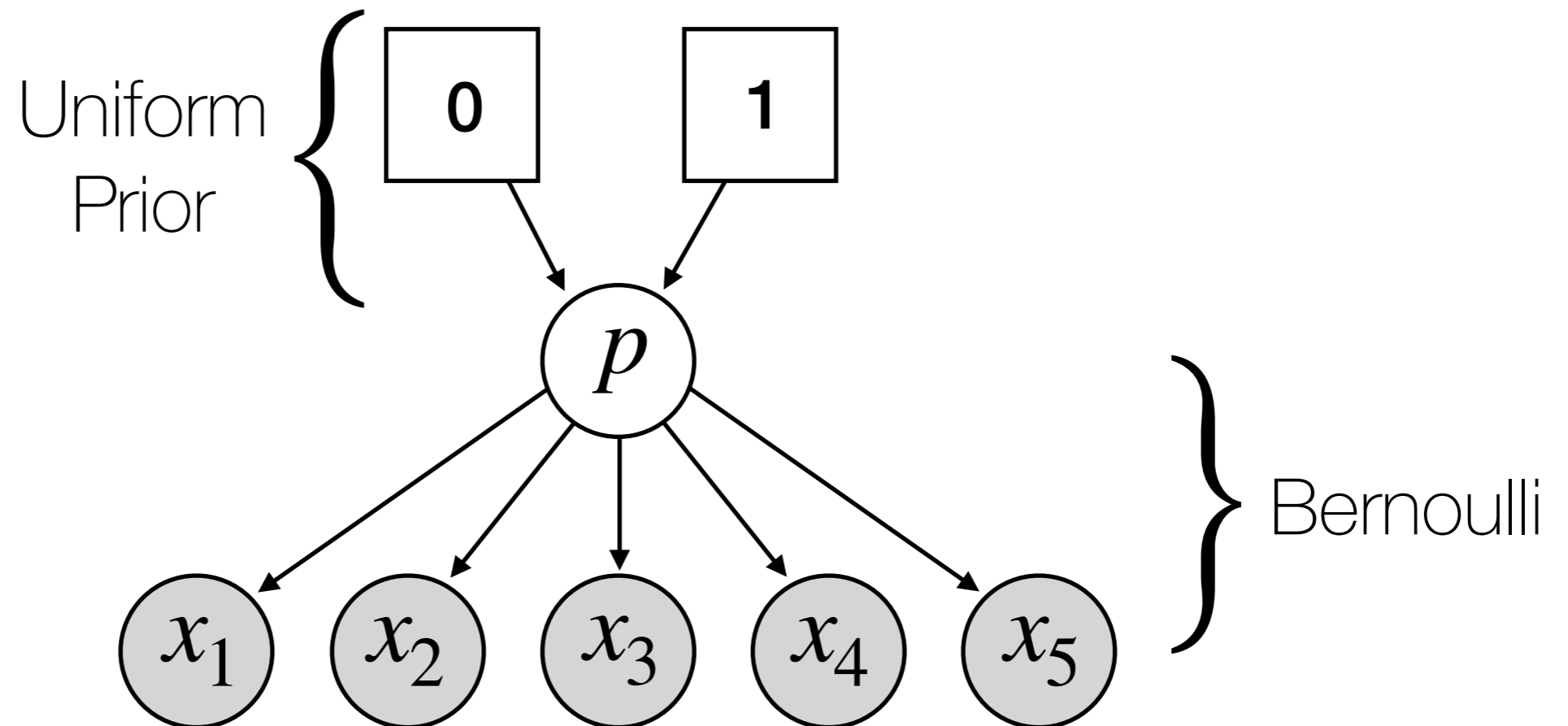
What's the structure of this model?
What can we infer about this model?
How much information will we have?

Single Bernoulli Trial



```
p ~ dnUnif(0, 1)
x ~ dnBernoulli(p)
x.clamp(0)
```

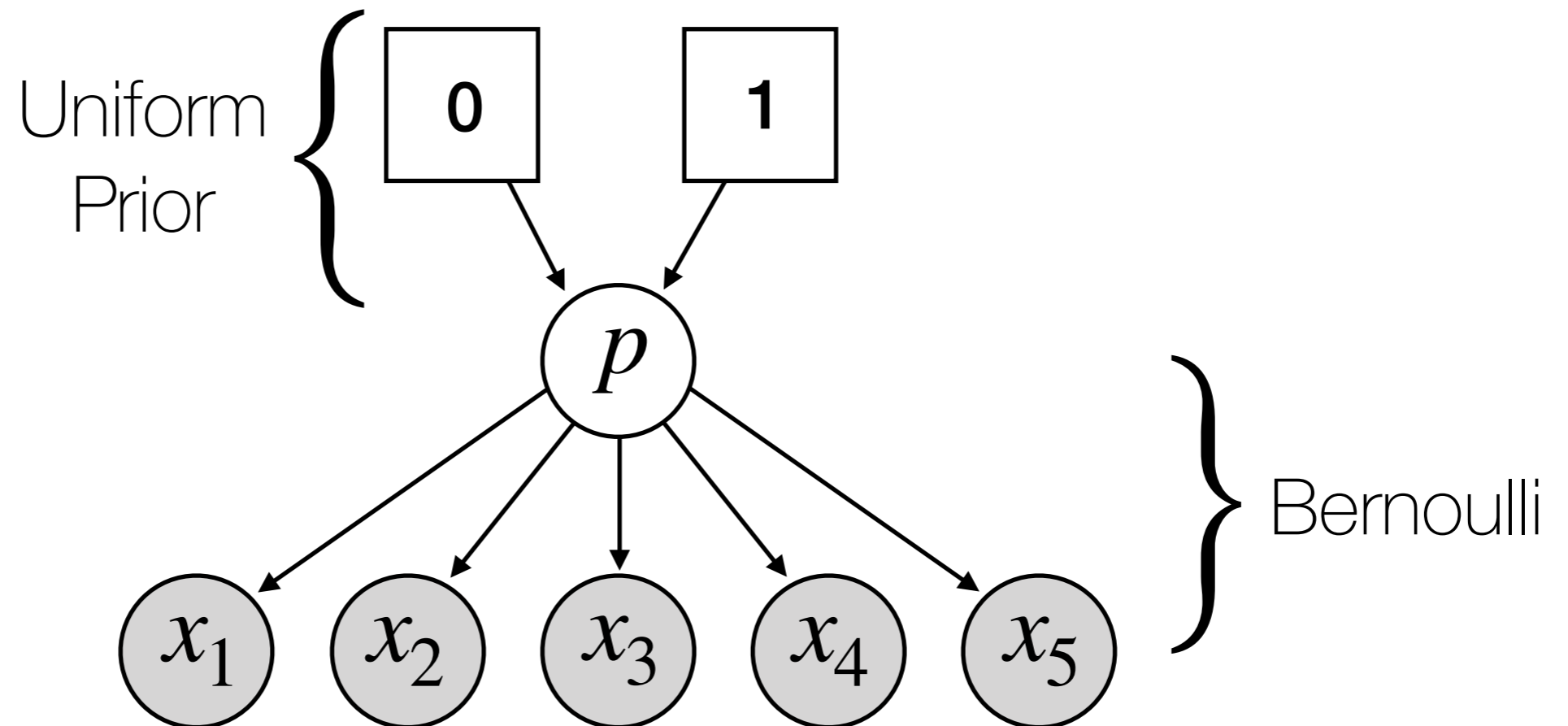
Series of Bernoullis with Linked p



What can we infer about this model?

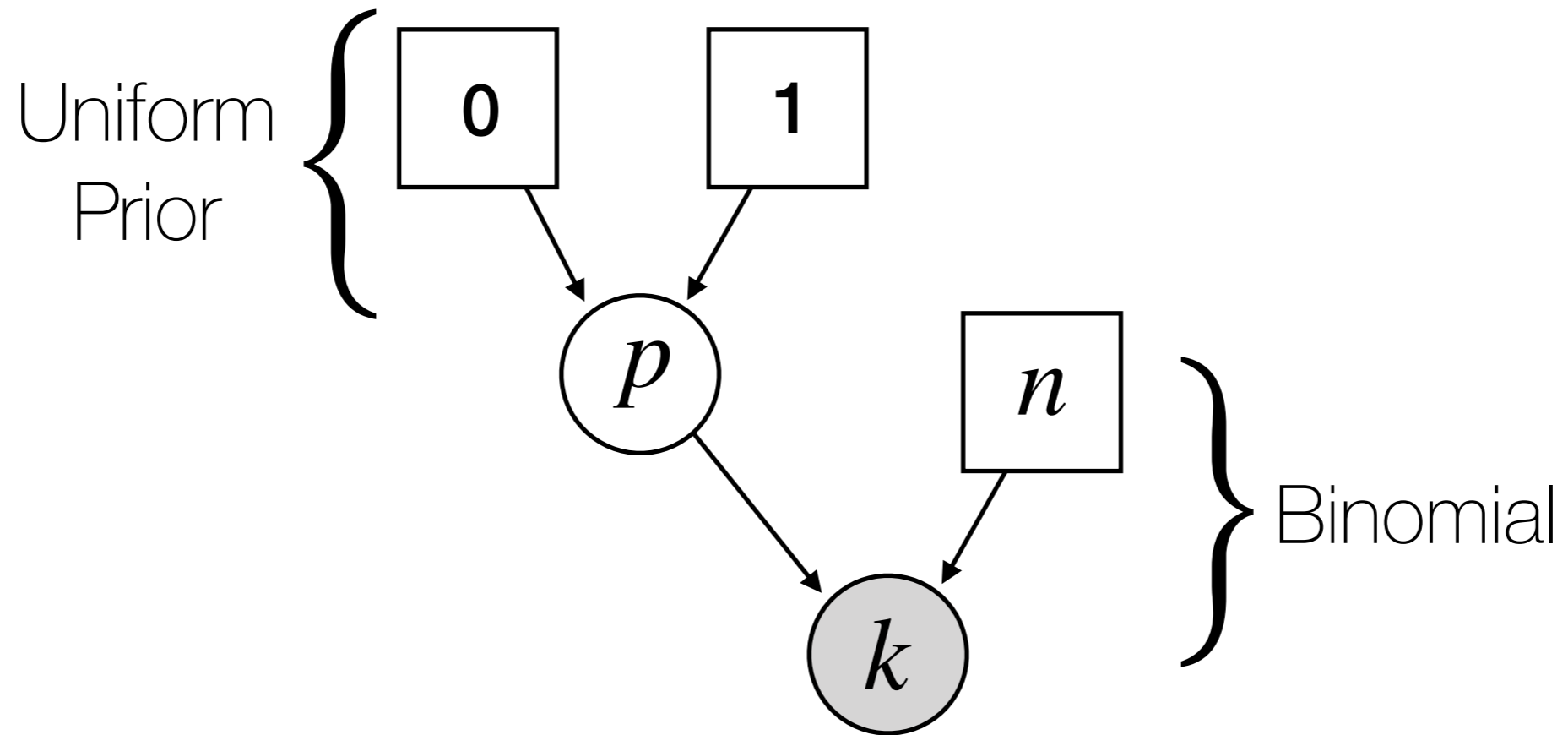
How much information will we have (relative to a single Bernoulli)?

Series of Bernoullis with Linked p



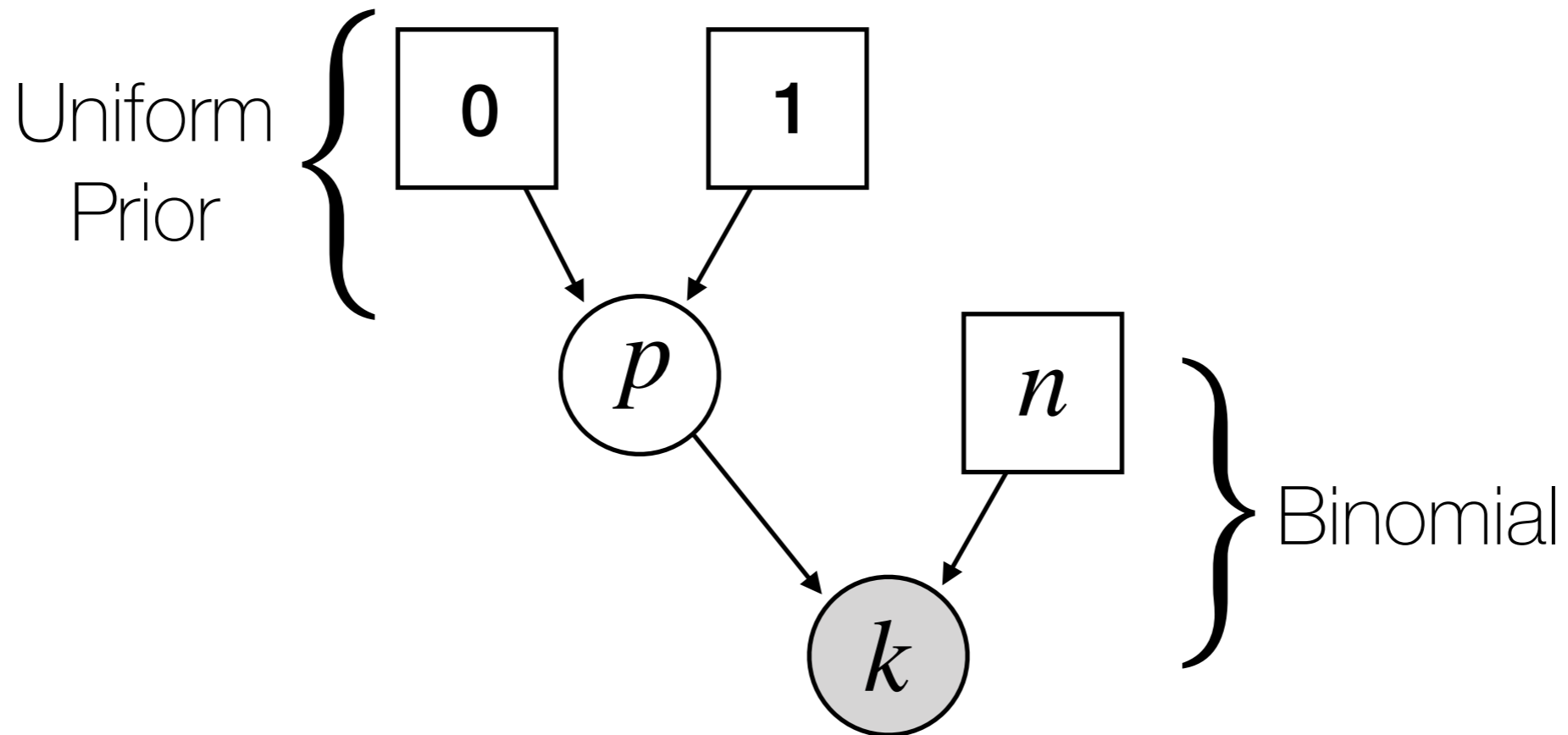
```
p ~ dnUnif(0,1)
for (i in 1:5){x[i] ~ dnBernoulli(p)}
x[1].clamp(0)
x[2].clamp(1)
x[3].clamp(1)
x[4].clamp(0)
x[5].clamp(1)
```

Single Binomial



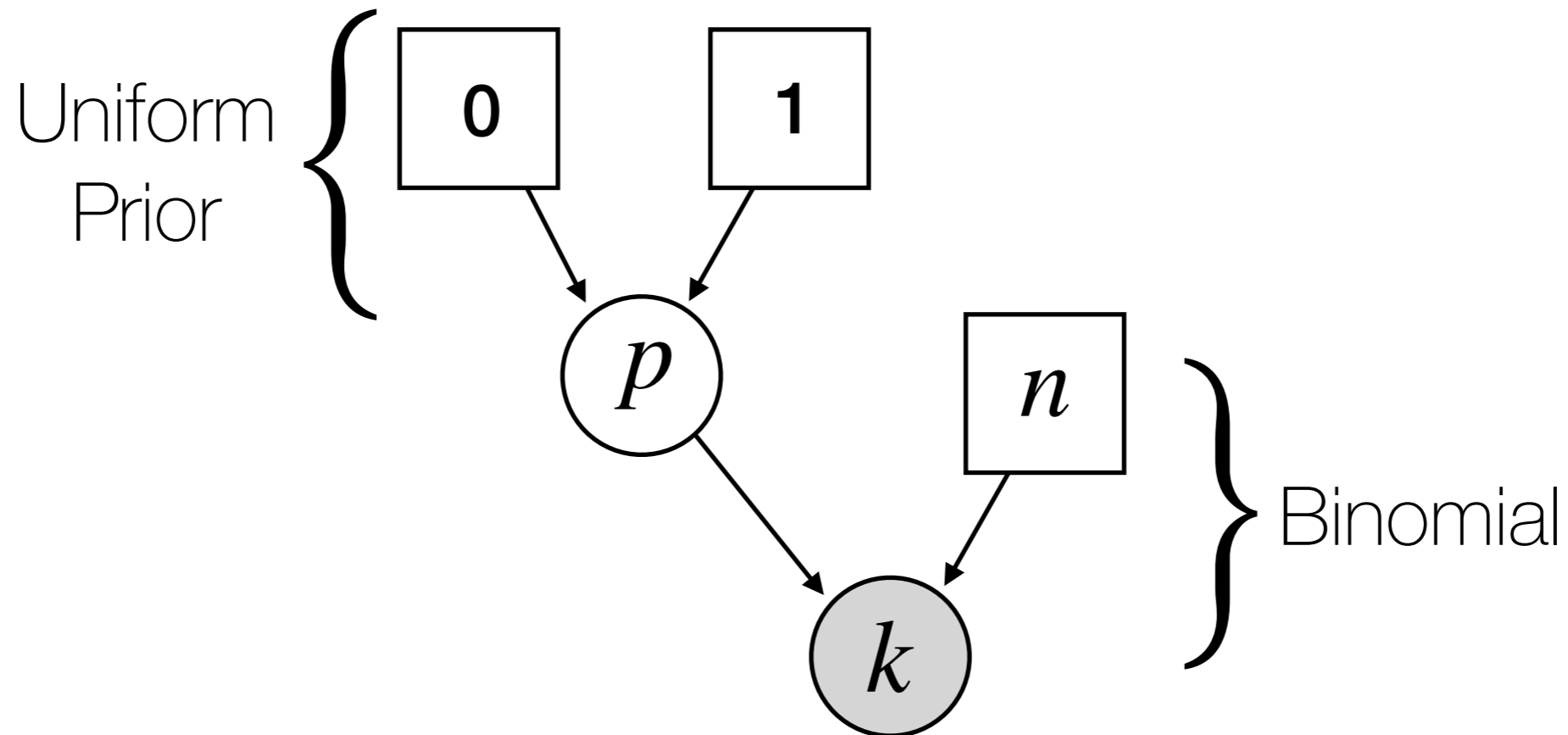
How does this model relate to the Bernoulli models?

Single Binomial



```
p ~ dnUnif(0, 1)
n <- 5
k ~ dnBinomial(n, p)
k.clamp(3)
```

Setting up MCMC in RevBayes

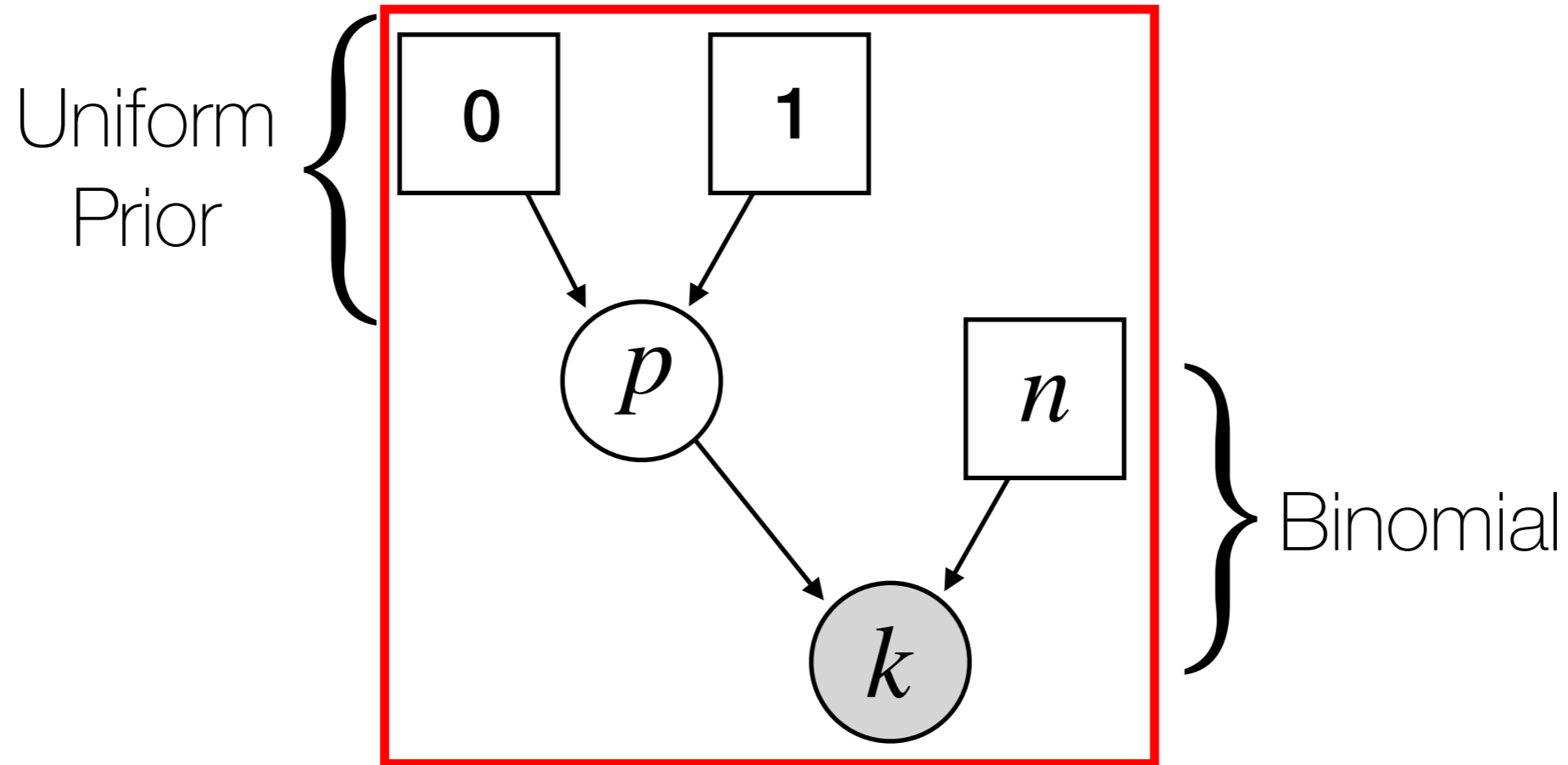


First, create a model object. You can pass any of the nodes to the constructor as a “handle”.

```
myModel = model(n)
```

NOTE: We use the = assignment operator for “workspace” variables.

Setting up MCMC in RevBayes

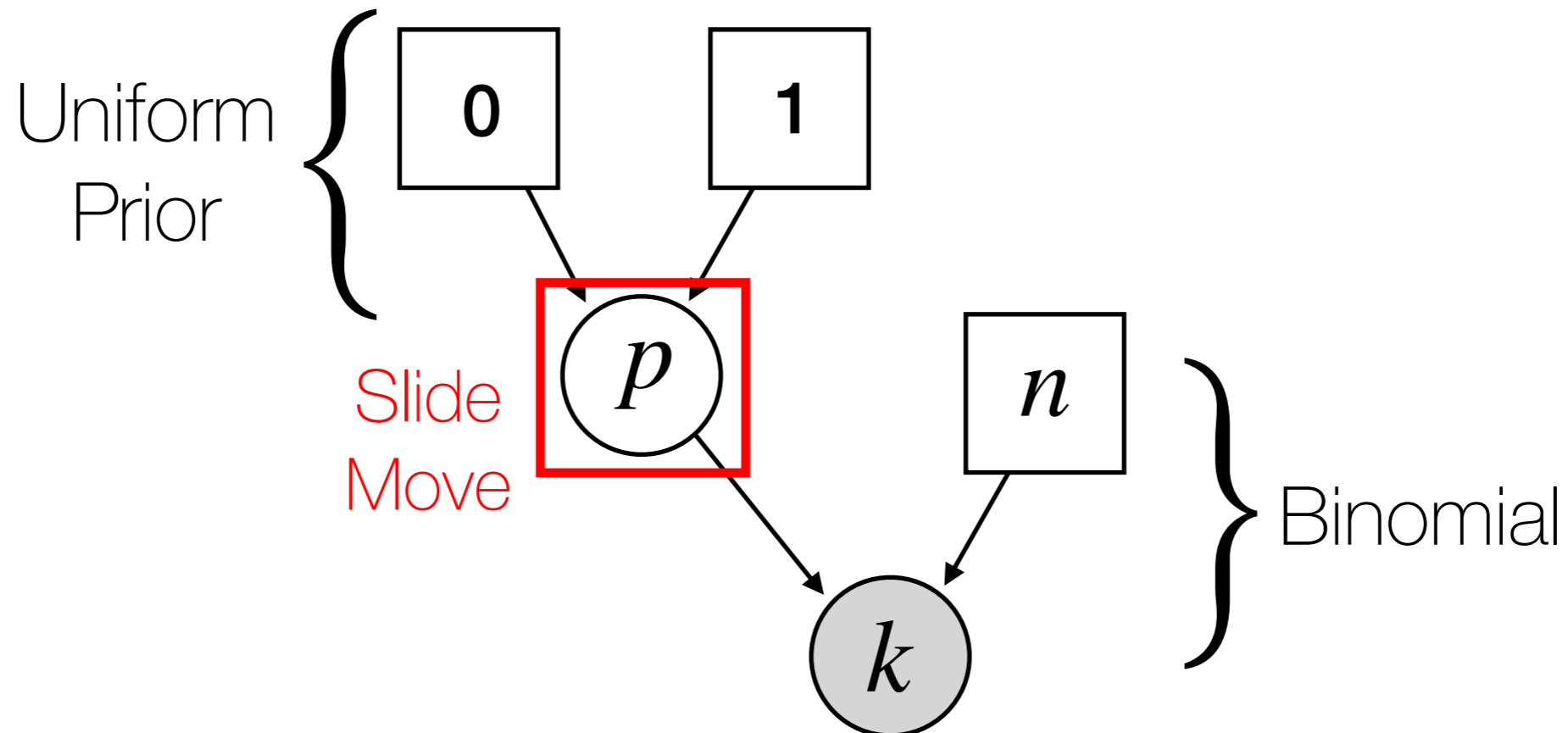


First, create a model object. You can pass any of the nodes to the constructor as a “handle”.

```
myModel = model(n)
```

NOTE: We use the = assignment operator for “workspace” variables.

Setting up MCMC in RevBayes



Next, we need to define a proposal distribution (move) for any parameters we are trying to infer.

```
moves = VectorMoves()  
moves.append( mvSlide(p,delta=0.1,weight=1) )
```

Many different move types are available in RevBayes.

Setting up MCMC in RevBayes

As our MCMC runs, we need to keep track of our progress and sampled parameter values. To do that we use monitors.

```
monitors = VectorMonitors()
monitors.append( mnScreen(printgen=1000,p) )
monitors.append( mnModel(filename="myMCMC.log", printgen=10) )
```

Running MCMC simulation

This simulation runs 1 independent replicate.

The simulator uses 1 different moves in a random move schedule with 1 moves per iteration

Iter	Posterior	Likelihood	Prior	p	elapsed	ETA
0	-10.2115	-10.2115	0	0.08309243	00:00:00	--:--:--
1000	-4.8278	-4.8278	0	0.5133643	00:00:00	--:--:--
2000	-4.97547	-4.97547	0	0.453571	00:00:00	00:00:00
3000	-4.78164	-4.78164	0	0.5619349	00:00:00	00:00:00
4000	-5.18177	-5.18177	0	0.7290954	00:00:00	00:00:00
5000	-5.98066	-5.98066	0	0.2898232	00:00:00	00:00:00
6000	-4.78464	-4.78464	0	0.5540578	00:00:00	00:00:00
7000	-4.78044	-4.78044	0	0.5690397	00:00:00	00:00:00
8000	-4.79273	-4.79273	0	0.541859	00:00:00	00:00:00
9000	-4.96352	-4.96352	0	0.4572184	00:00:00	00:00:00
10000	-4.80438	-4.80438	0	0.6120314	00:00:00	00:00:00

Setting up MCMC in RevBayes

Next we create an MCMC object.

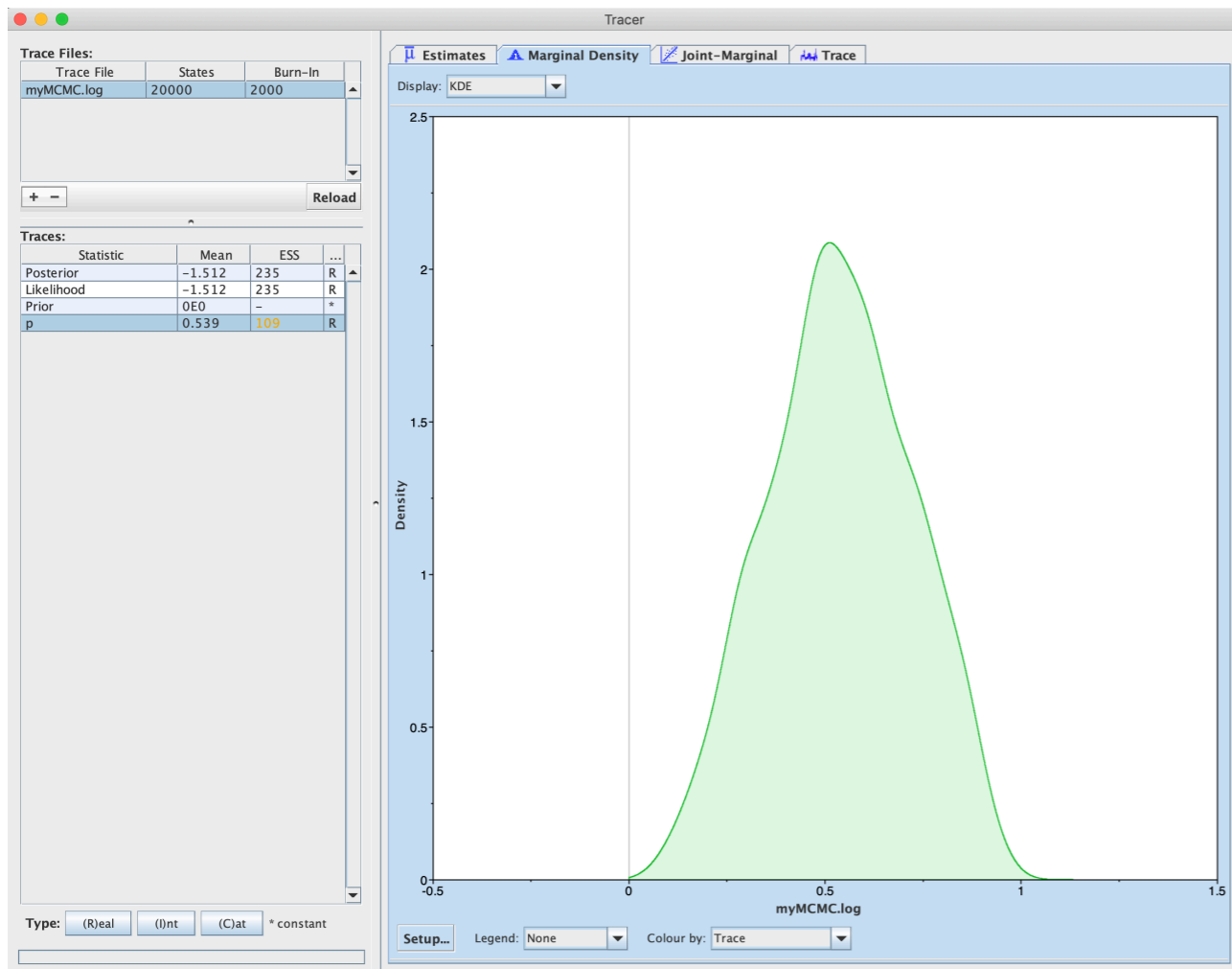
```
myMCMC = mcmc(myModel,moves,monitors)
```

Now, we start the MCMC!

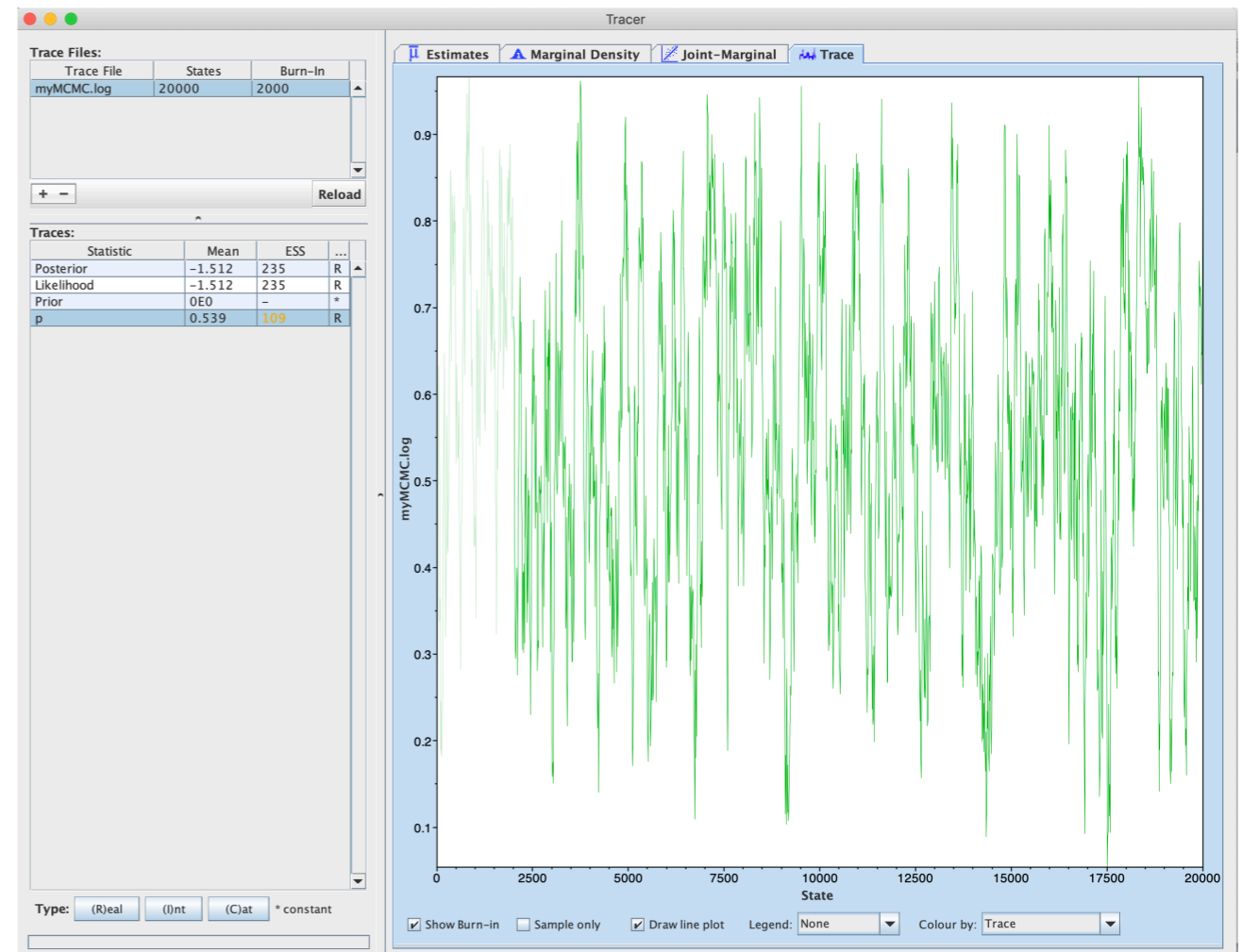
```
myMCMC.run(20000)
```

Wait - what does this 20,000 mean?

Tracer

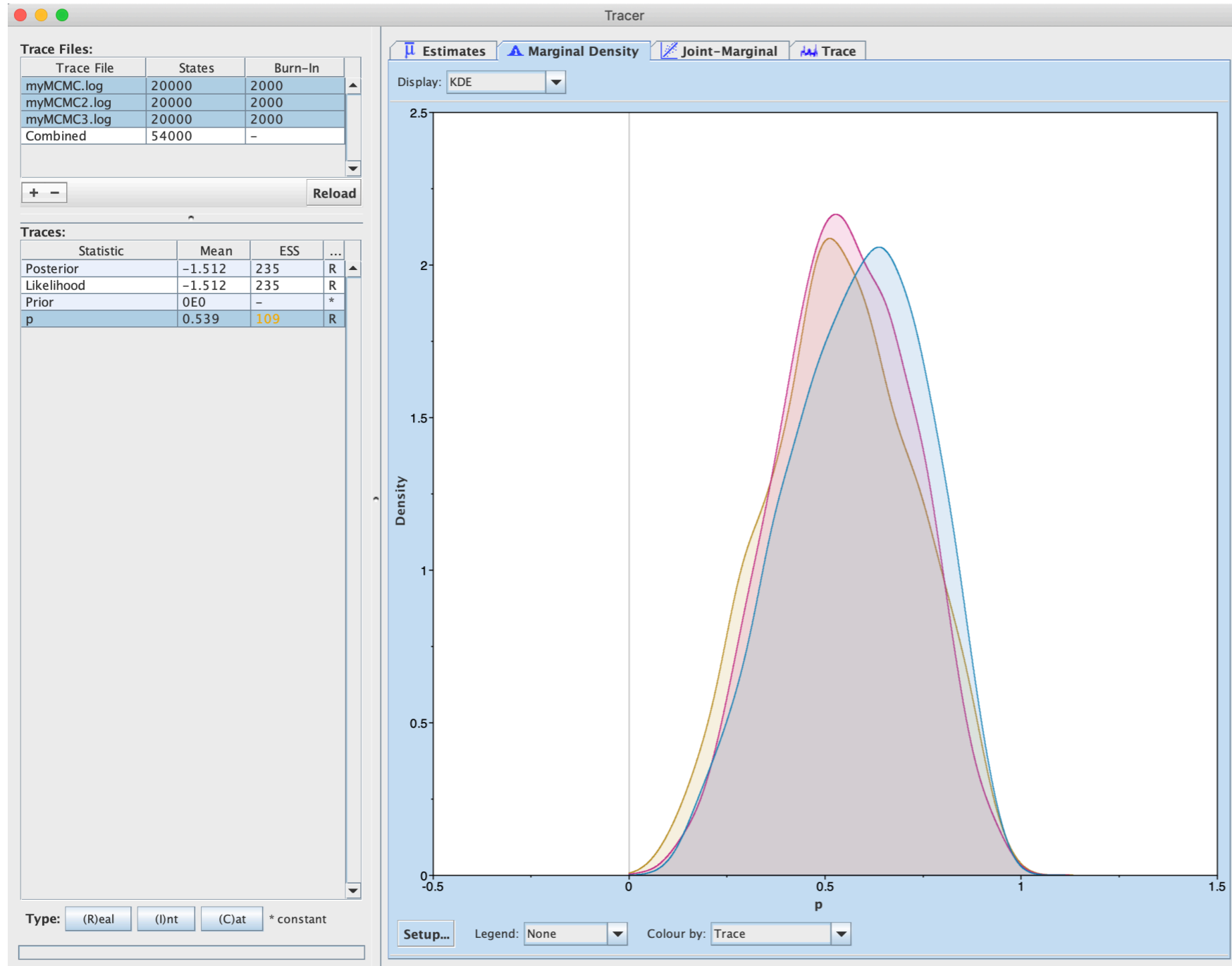


Marginal Distribution



Trace Plot

Assessing Convergence



Traces:

Statistic	Mean	ESS	...
Posterior	-1.512	235	R
Likelihood	-1.512	235	R
Prior	0E0	-	*
p	0.539	109	R

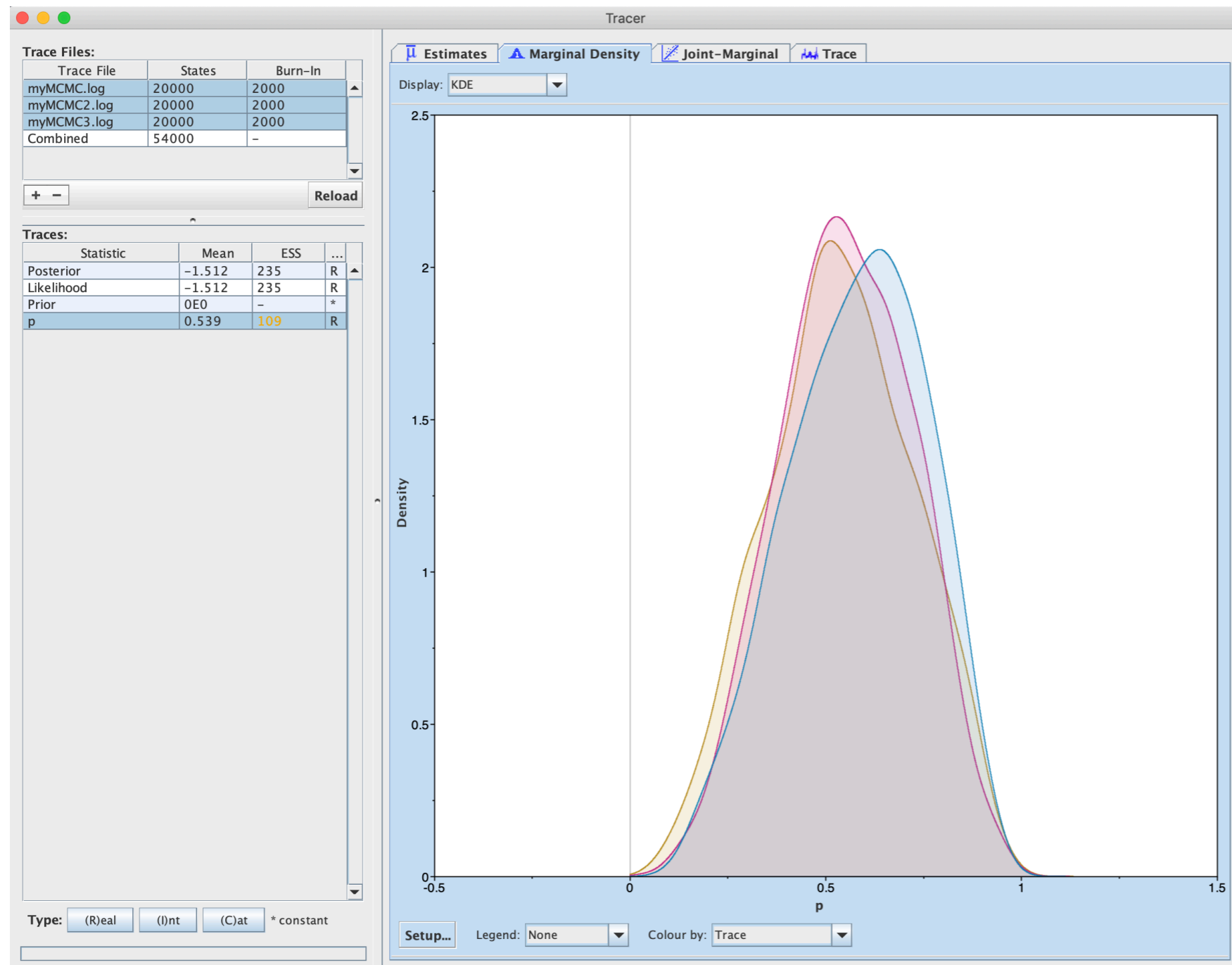
Traces:

Statistic	Mean	ESS	...
Posterior	-1.427	282	R
Likelihood	-1.427	282	R
Prior	0E0	-	*
p	0.551	94	R

Traces:

Statistic	Mean	ESS	...
Posterior	-1.437	222	R
Likelihood	-1.437	222	R
Prior	0E0	-	*
p	0.58	104	R

Assessing Convergence



Statistic	Mean	ESS	...
Posterior	-1.512	235	R
Likelihood	-1.512	235	R
Prior	0E0	-	*
p	0.539	109	R

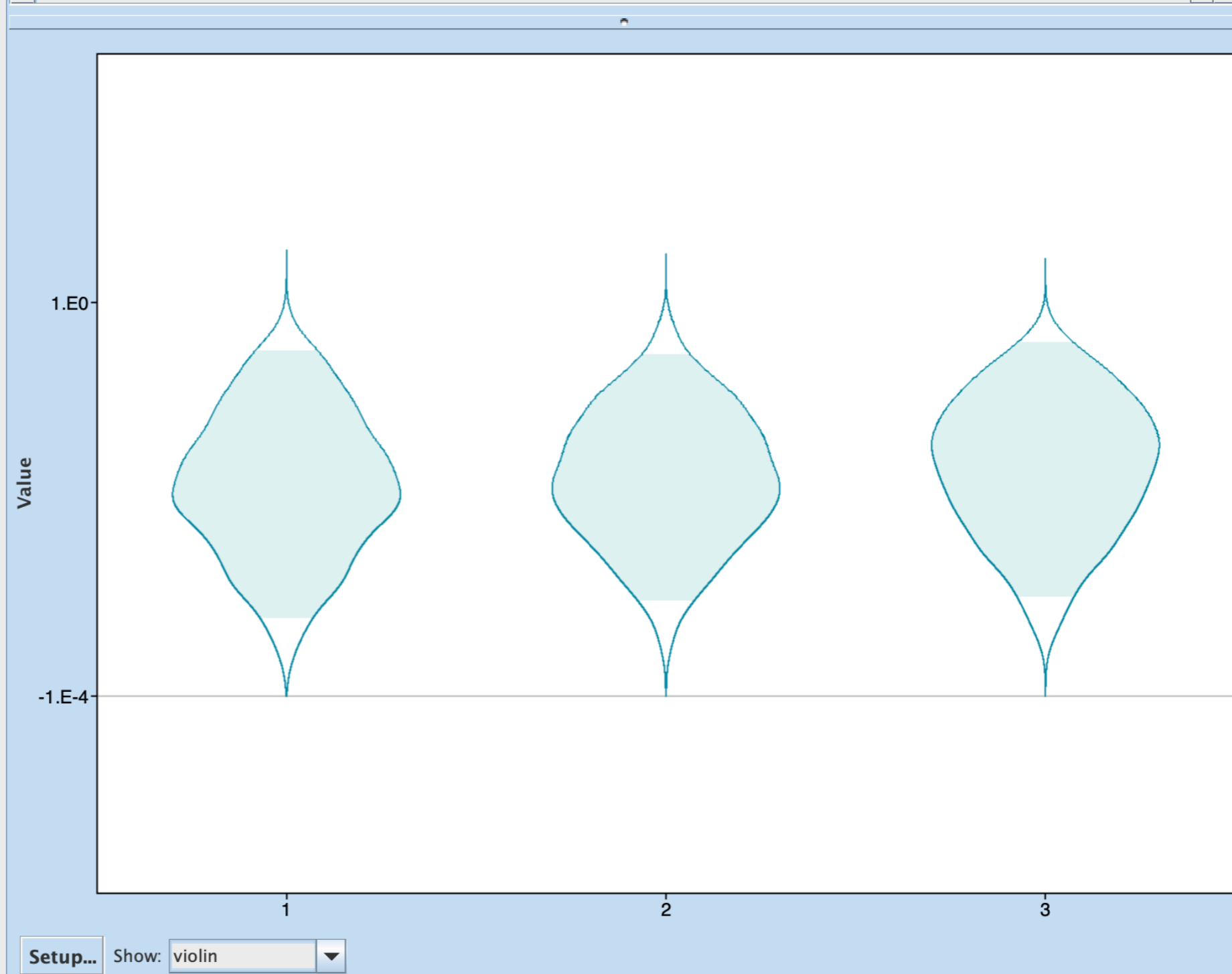
Statistic	Mean	ESS	...
Posterior	-1.427	282	R
Likelihood	-1.427	282	R
Prior	0E0	-	*
p	0.551	94	R

Statistic	Mean	ESS	...
Posterior	-1.437	222	R
Likelihood	-1.437	222	R
Prior	0E0	-	*
p	0.58	104	R

How many samples did we draw during our MCMC analyses?

95% Highest Posterior Density (HPD) Intervals

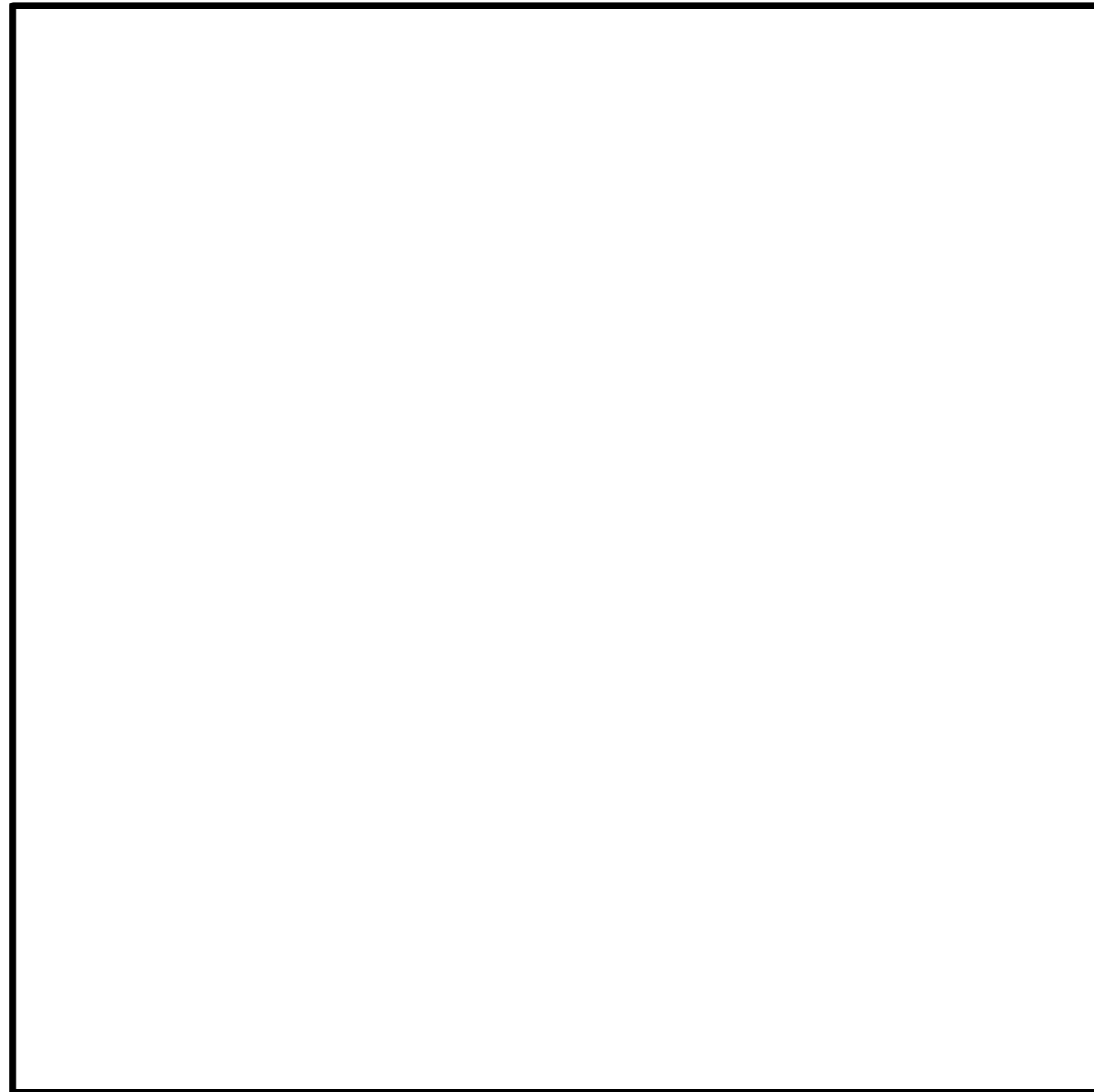
Summary Statistic	myMCMC.logp	myMCMC2.logp	myMCMC3.logp
mean	0.5389	0.5515	0.5805
stderr of mean	0.0175	0.0173	0.0171
stdev	0.1828	0.1681	0.1745
variance	0.0334	0.0283	0.0305
median	0.5357	0.5512	0.5908
value range	[0.0543, 0.9669]	[0.0437, 0.9709]	[0.0958, 0.9501]
geometric mean	0.5024	0.5219	0.5495
95% HPD interval	[0.1984, 0.878]	[0.2426, 0.8685]	[0.2526, 0.8996]
auto-correlation time (ACT)	164.7238	190.594	173.4795
effective sample size (ESS)	109.3	94.5	103.8
number of samples	1801	1801	1801



Phylogenetic Graphical Models

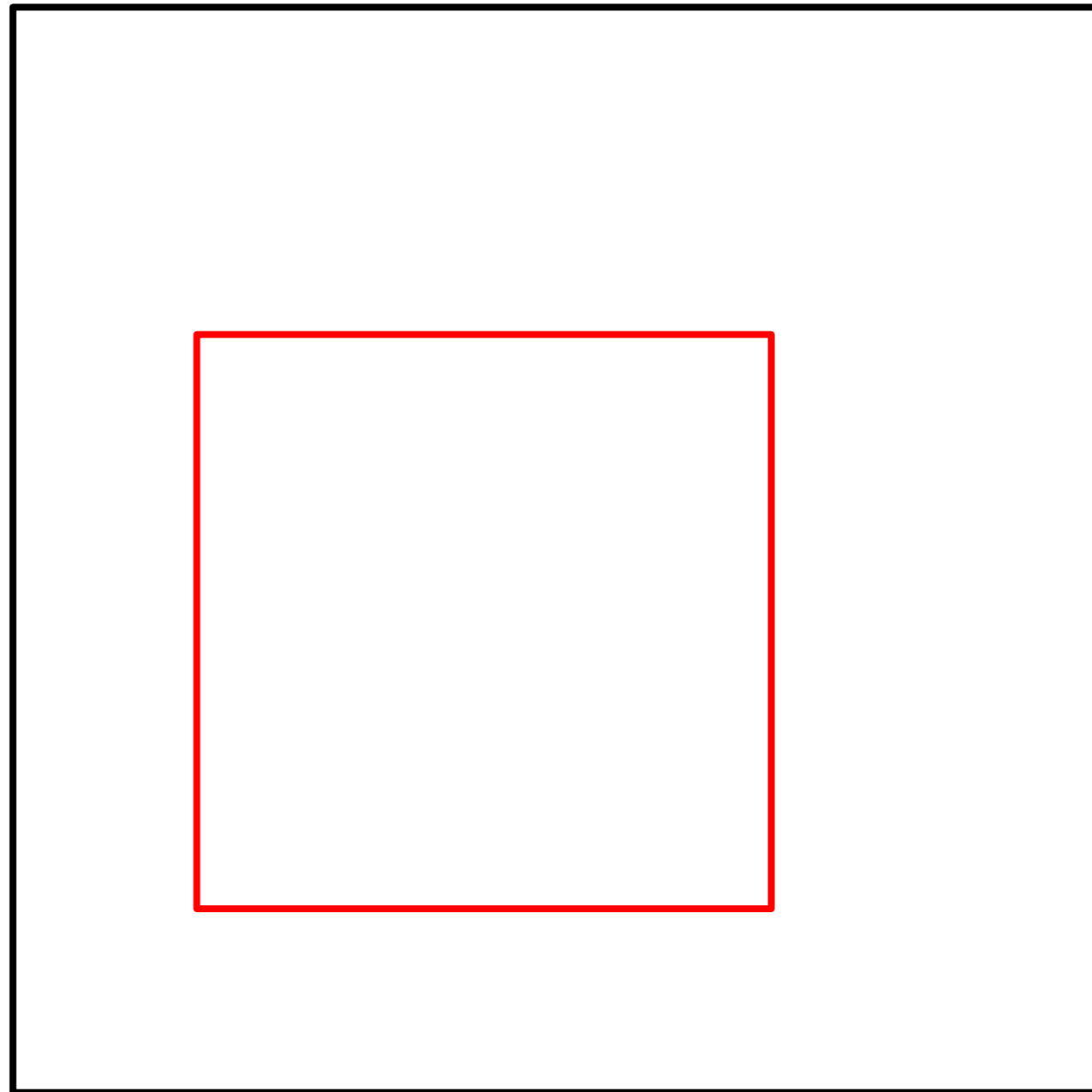
Model Space (conceptual)

All
Possible
Ways
Genes
Could
Evolve



Model Space (conceptual)

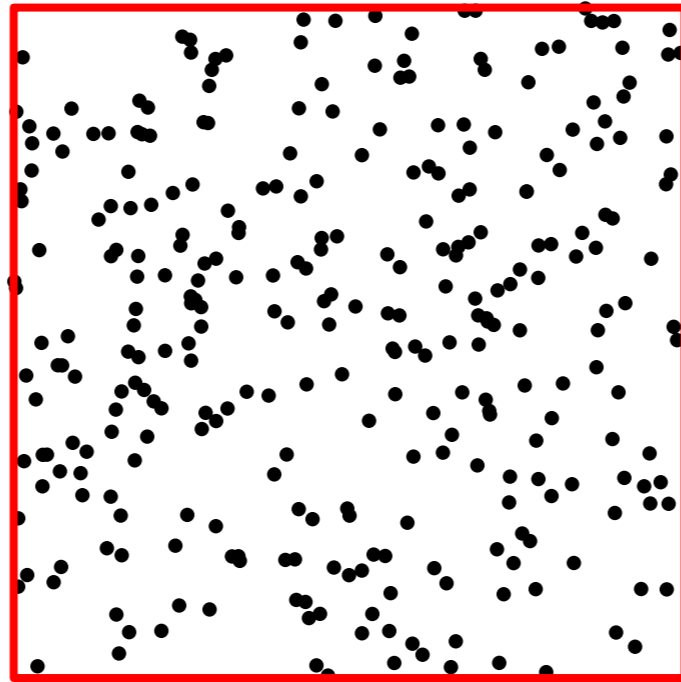
All
Possible
Ways
Genes
Could
Evolve



The
Types of
Evolution
That Are
Built Into
Many
Programs

Our Hope

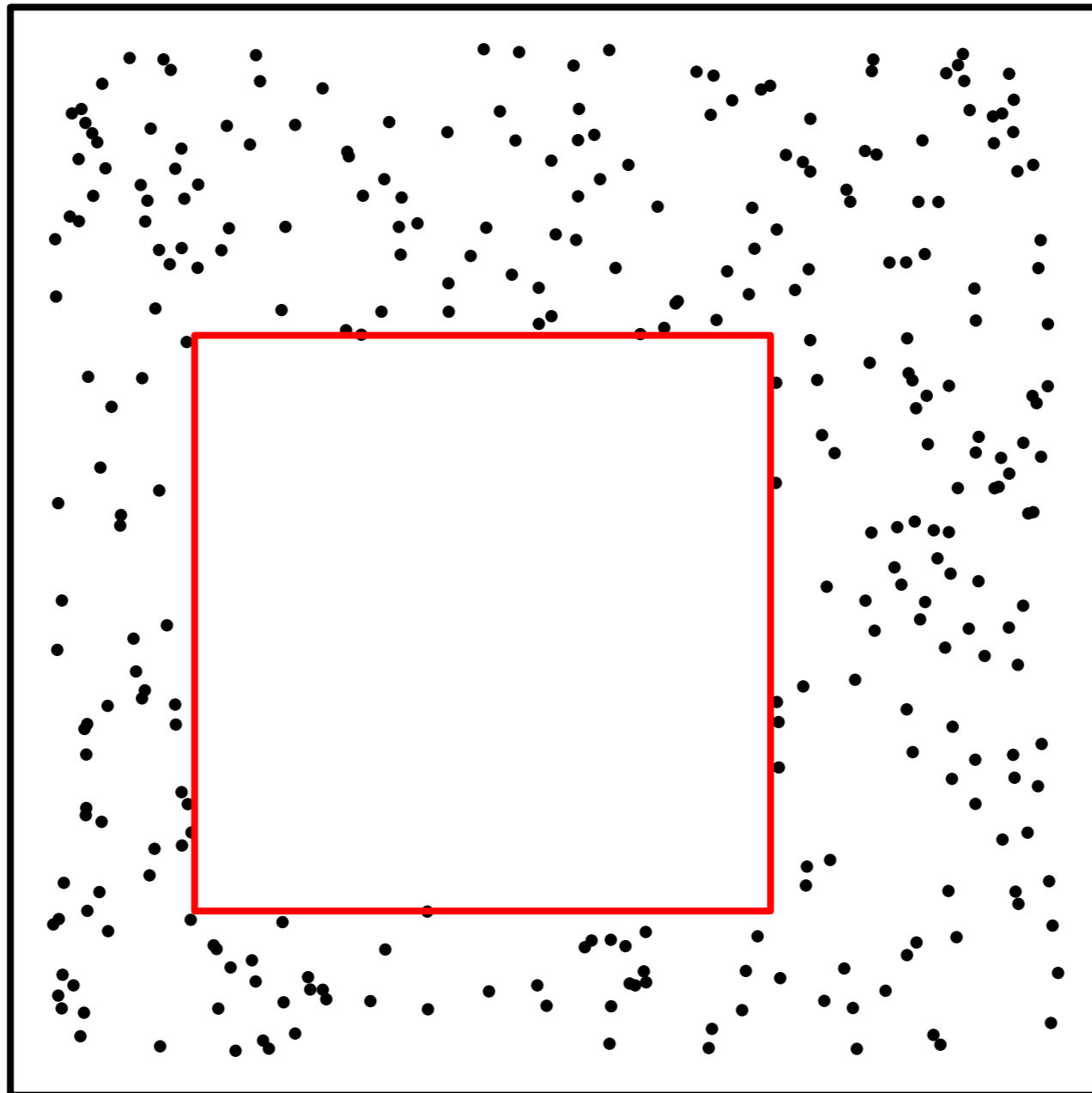
All
Possible
Ways
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Evolve



The
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Evolution
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Many
Programs

Our Fears

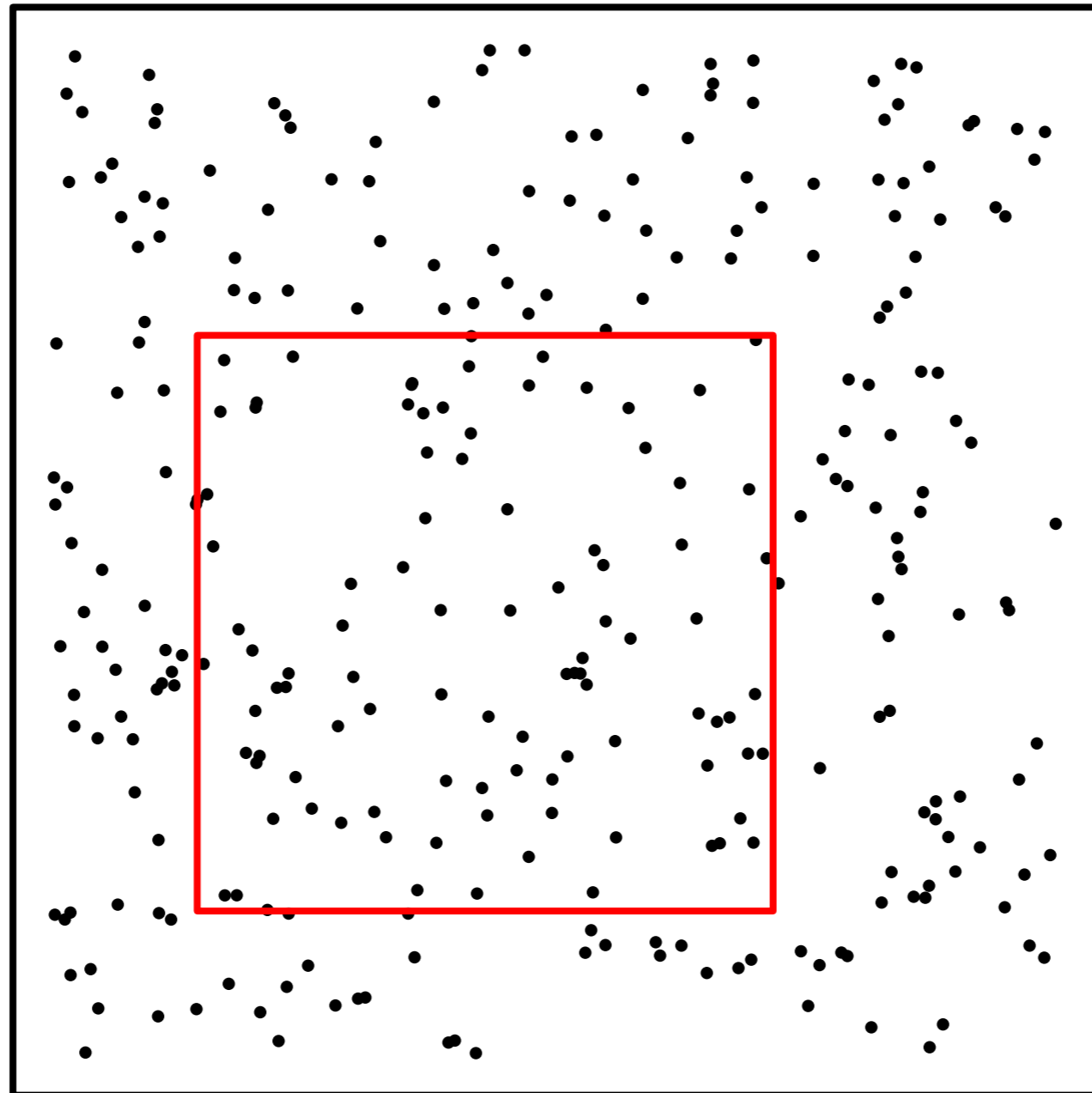
All
Possible
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The
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Evolution
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The Probable Truth

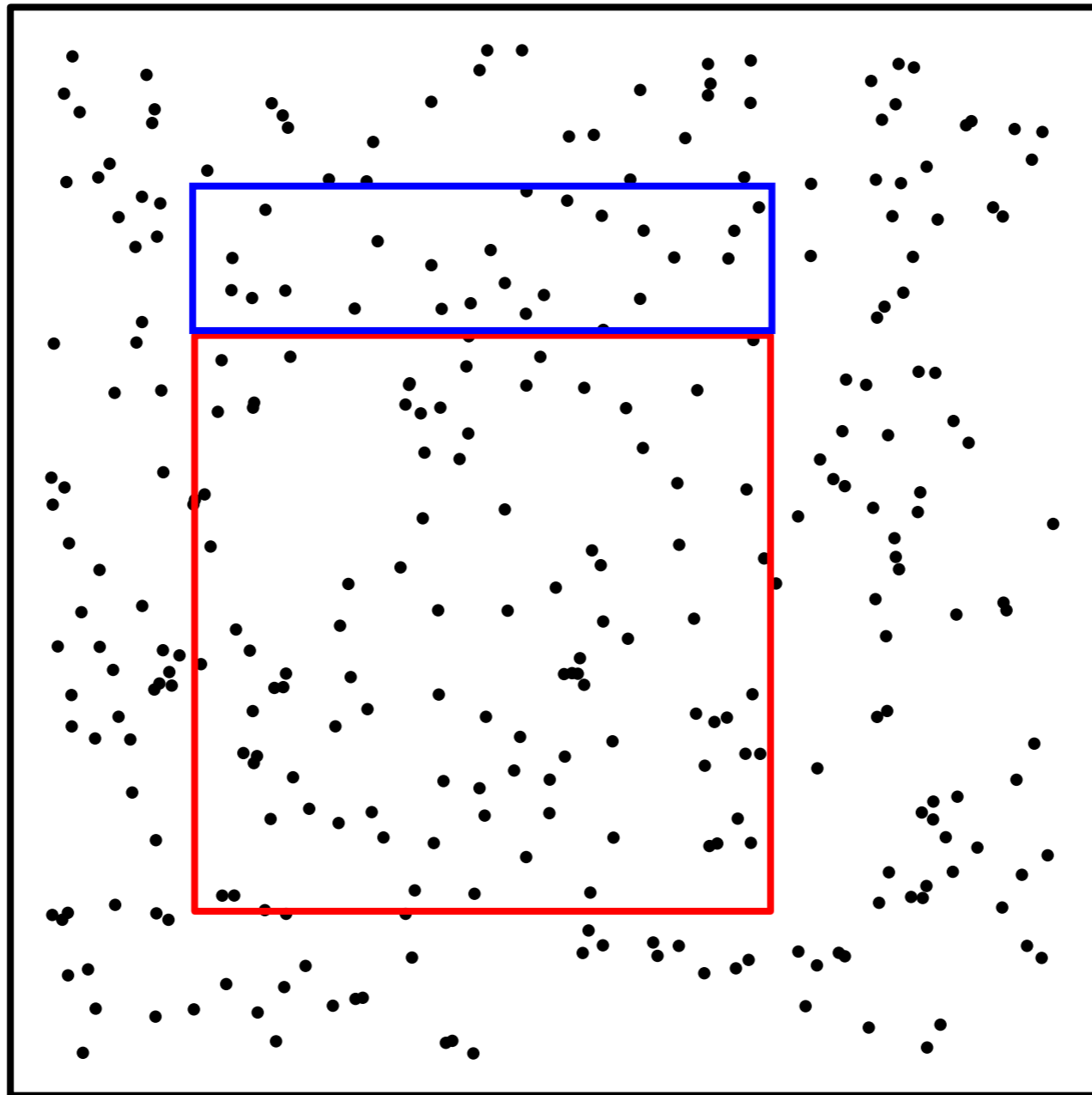
All
Possible
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The
Types of
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Programs

The Probable Truth

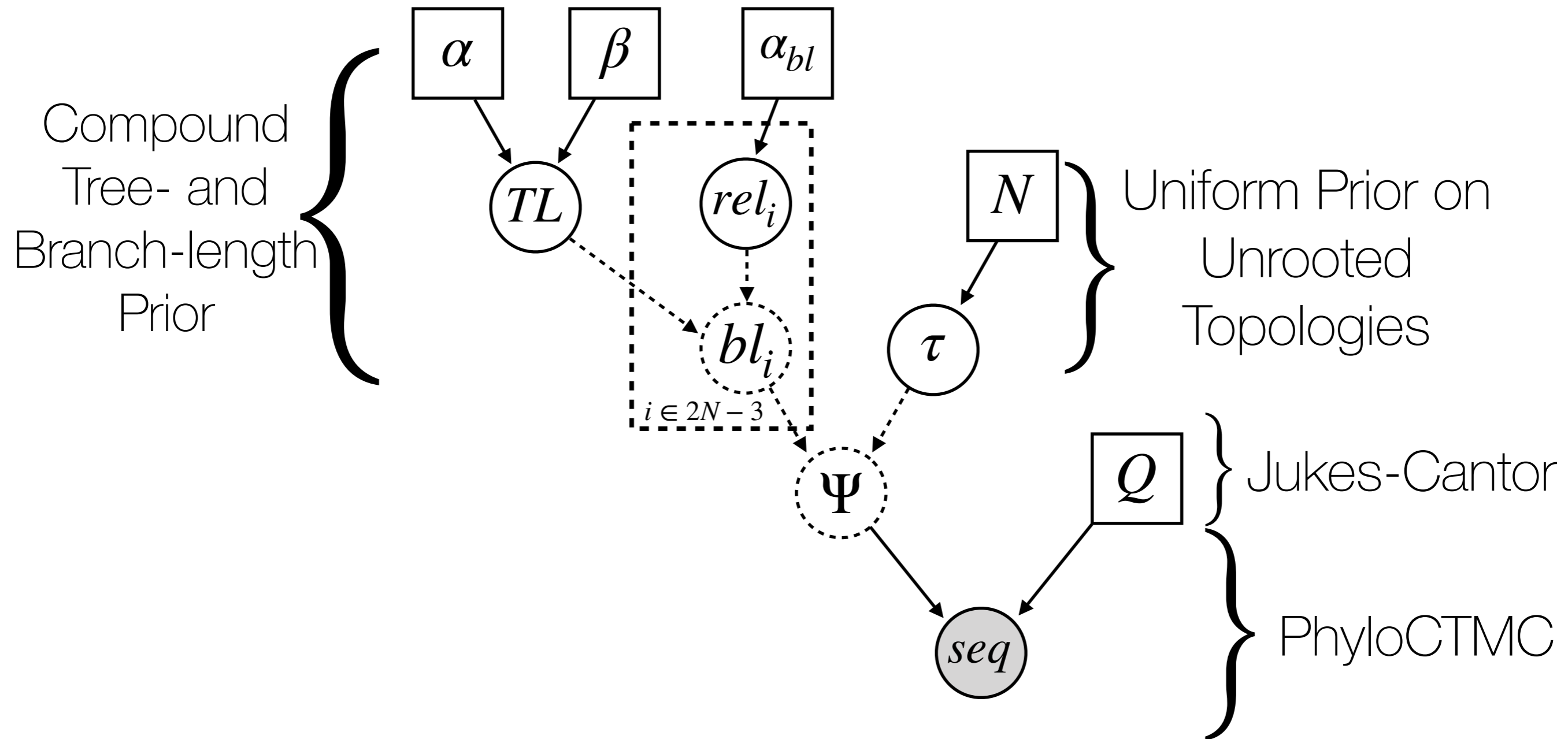
All
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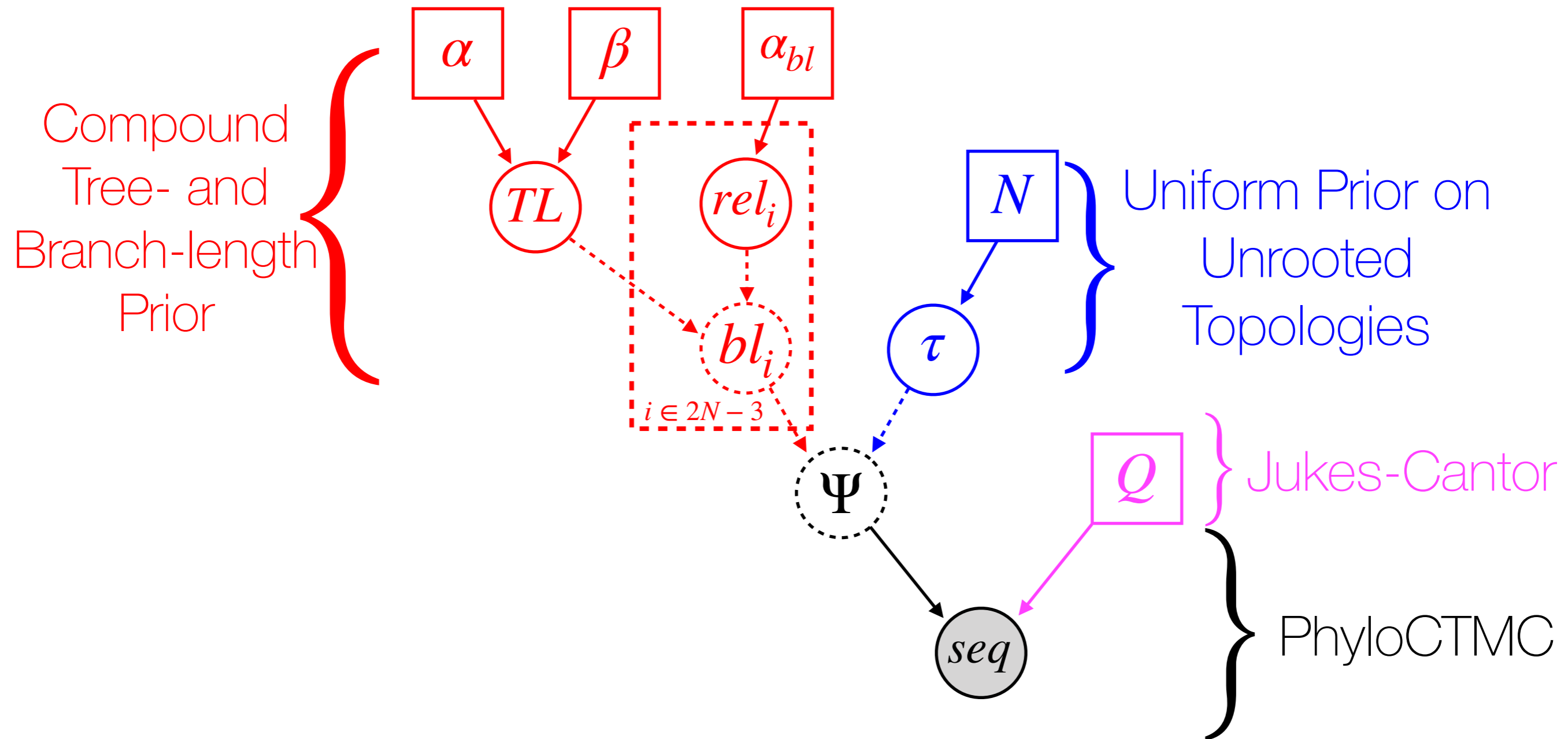
The
Types of
Evolution
That Are
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Many
Programs

Types of Evolution that You'd Like to Try!

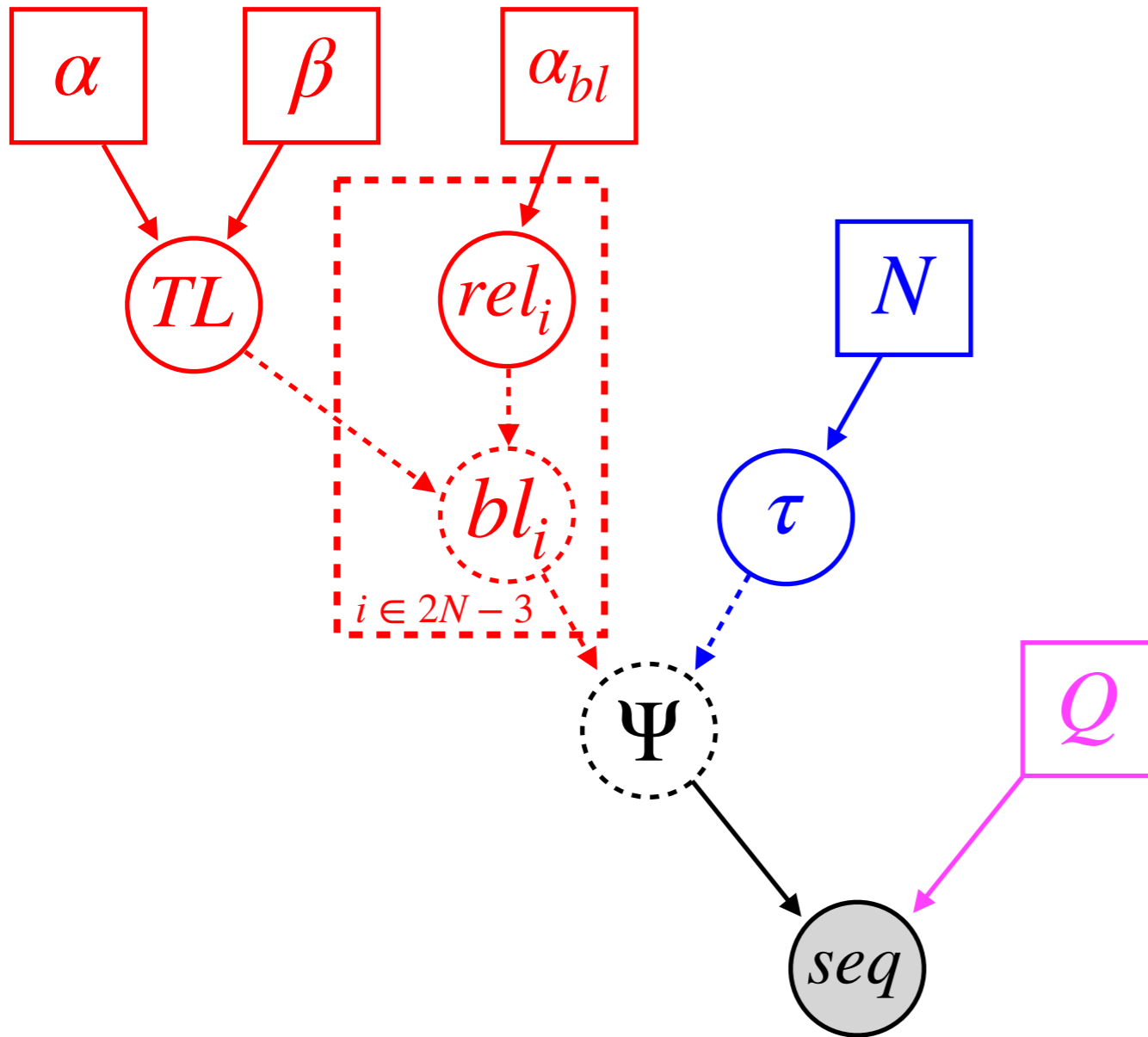
Jukes-Cantor



Jukes-Cantor



Jukes-Cantor



```
data = readDiscreteCharacterData("myData.nex")
taxa <- data.taxa()
n_taxa <- data.n_taxa()
n_branches <- 2 * n_taxa - 3
```

```
topology ~ dnUniformTopology(taxa)
```

```
alpha <- 2
beta <- 4
```

```
TL ~ dnGamma(alpha, beta)
```

```
alpha_bl <- 1.0
```

```
rel_branch_lengths ~ dnDirichlet( rep(alpha_bl, n_branches) )
```

```
br_lens := rel_branch_lengths * TL
```

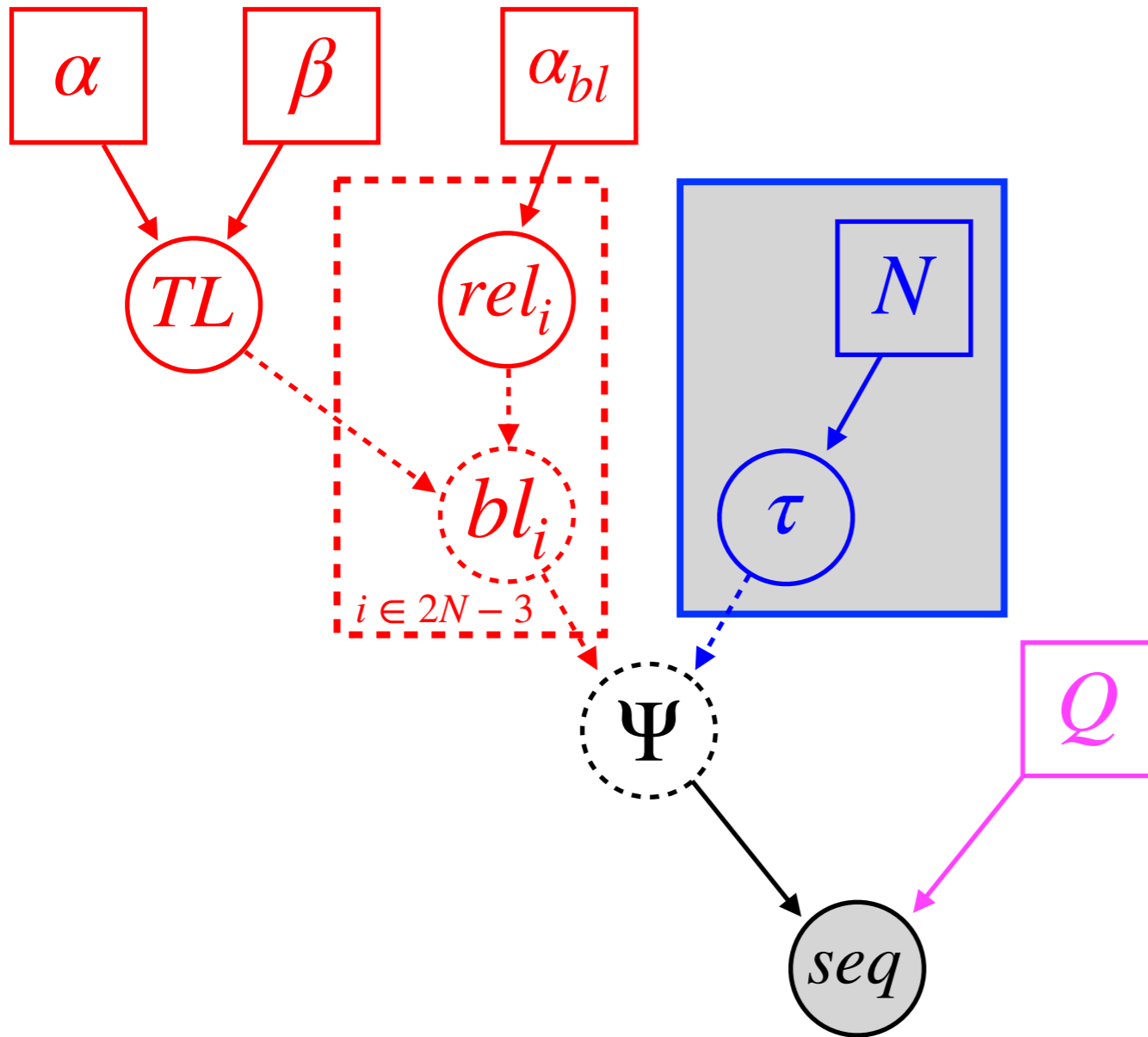
```
Q <- fnJC(4)
```

```
psi := treeAssembly(topology, br_lens)
```

```
seq ~ dnPhyloCTMC(tree=psi, Q=Q, type="DNA")
```

```
seq.clamp(data)
```

Jukes-Cantor



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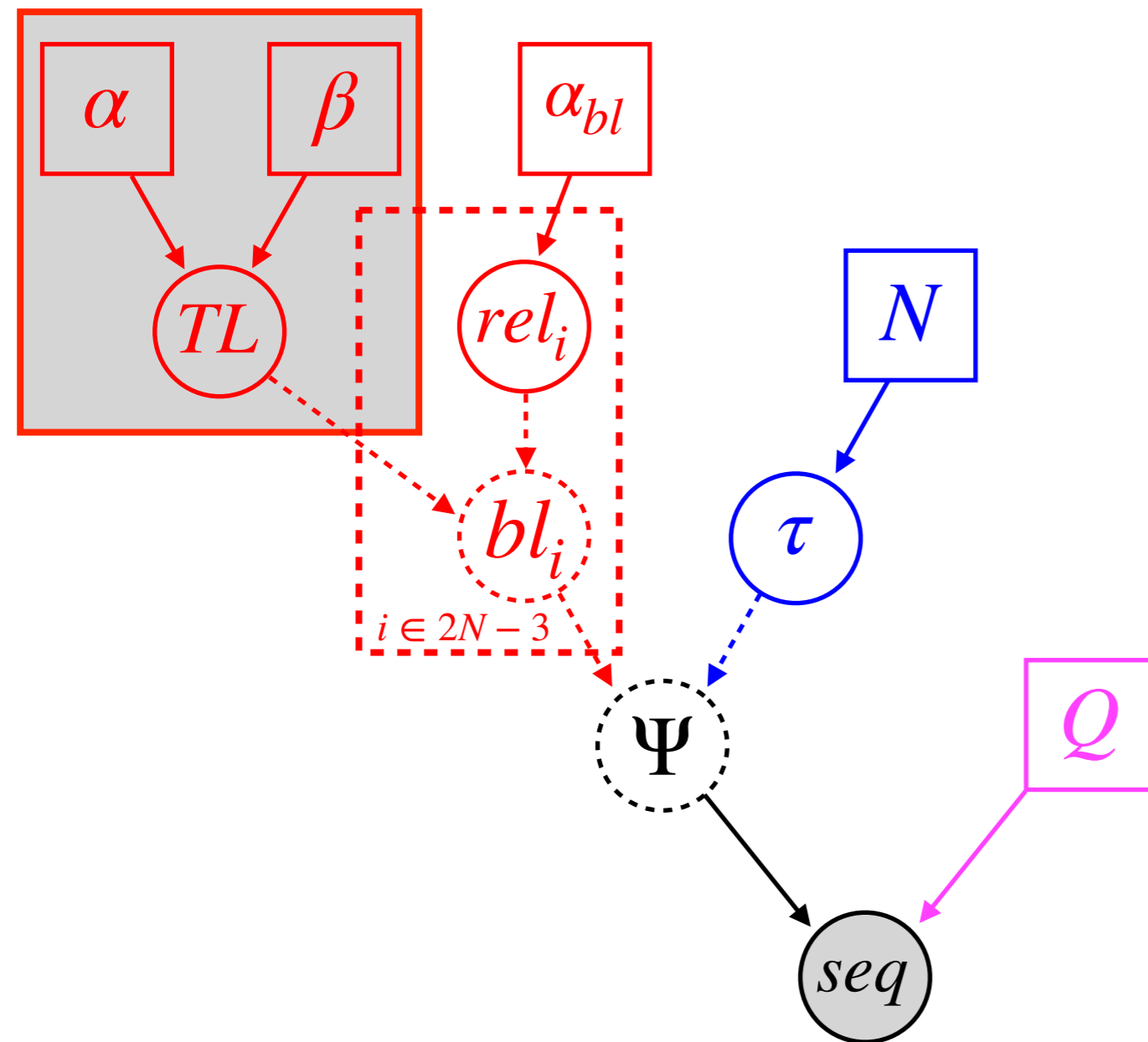
```

seq.clamp(data)

```

Jukes-Cantor

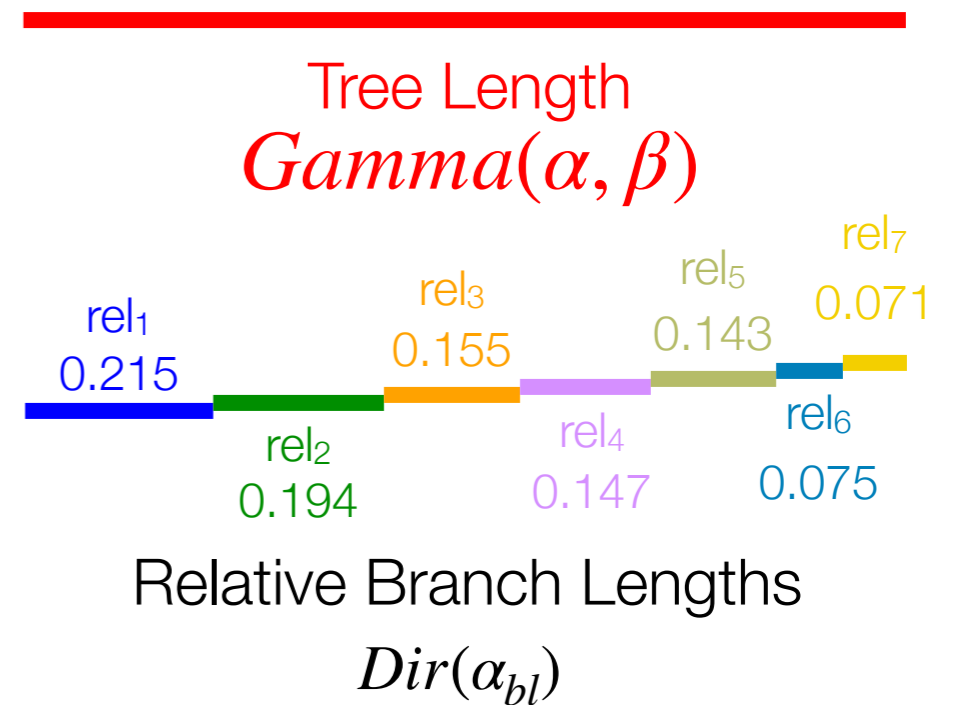
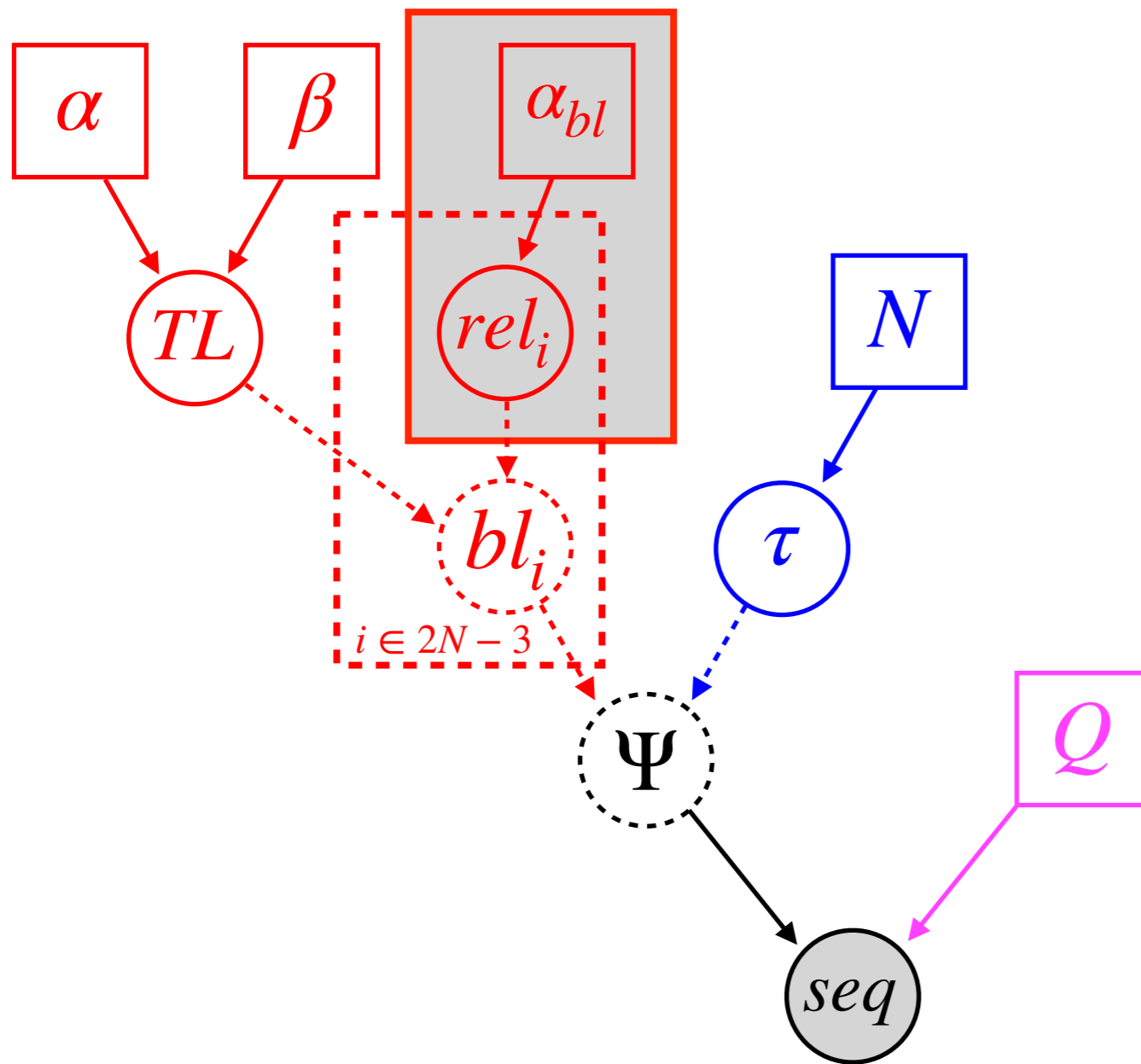
(Compound Dirichlet tree- and branch-length prior)



Tree Length
 $\text{Gamma}(\alpha, \beta)$

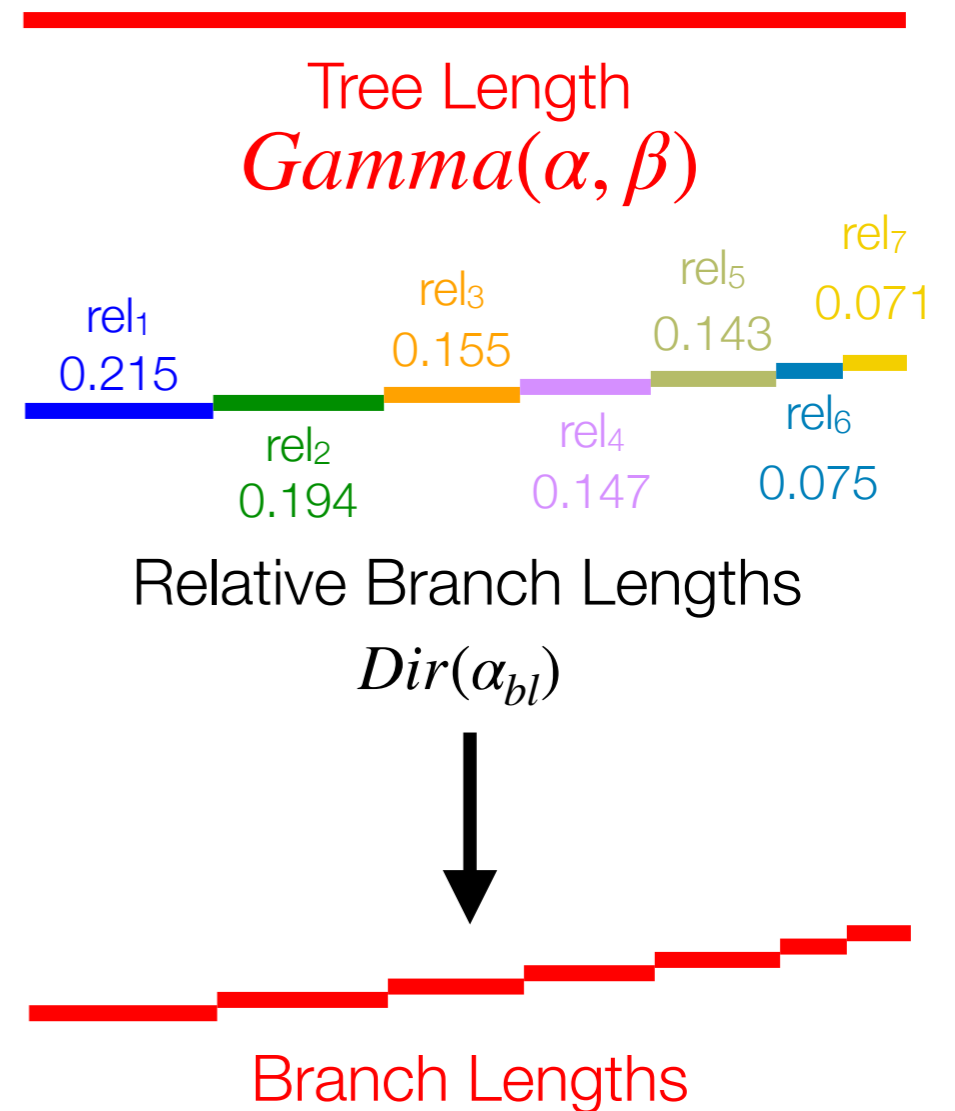
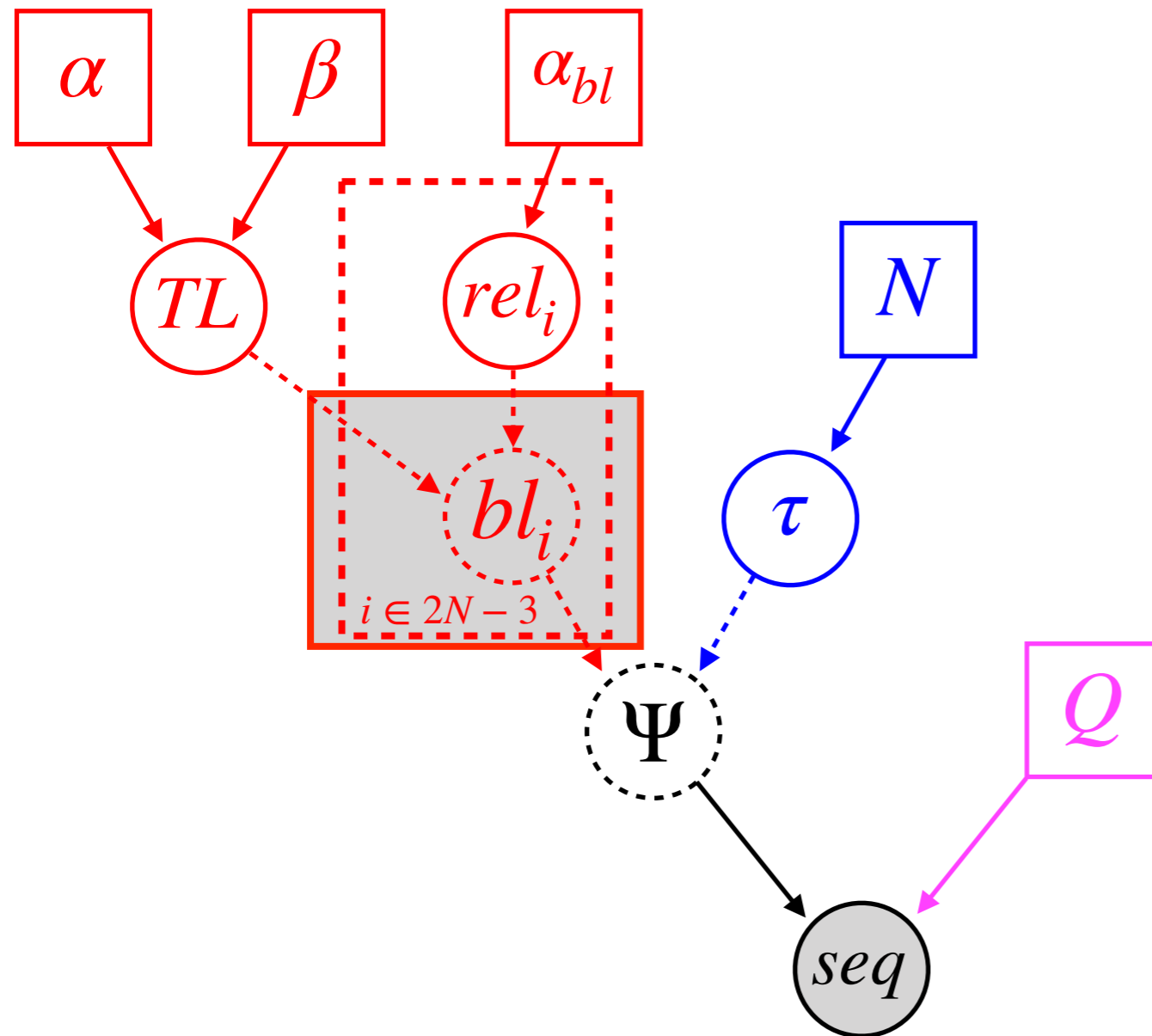
Jukes-Cantor

(Compound Dirichlet tree- and branch-length prior)



Jukes-Cantor

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Jukes-Cantor

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```
alpha <- 2
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```
alpha_bl <- 1.0
```

```
rel_branch_lengths ~ dnDirichlet(rep(alpha_bl,n_branches))
```

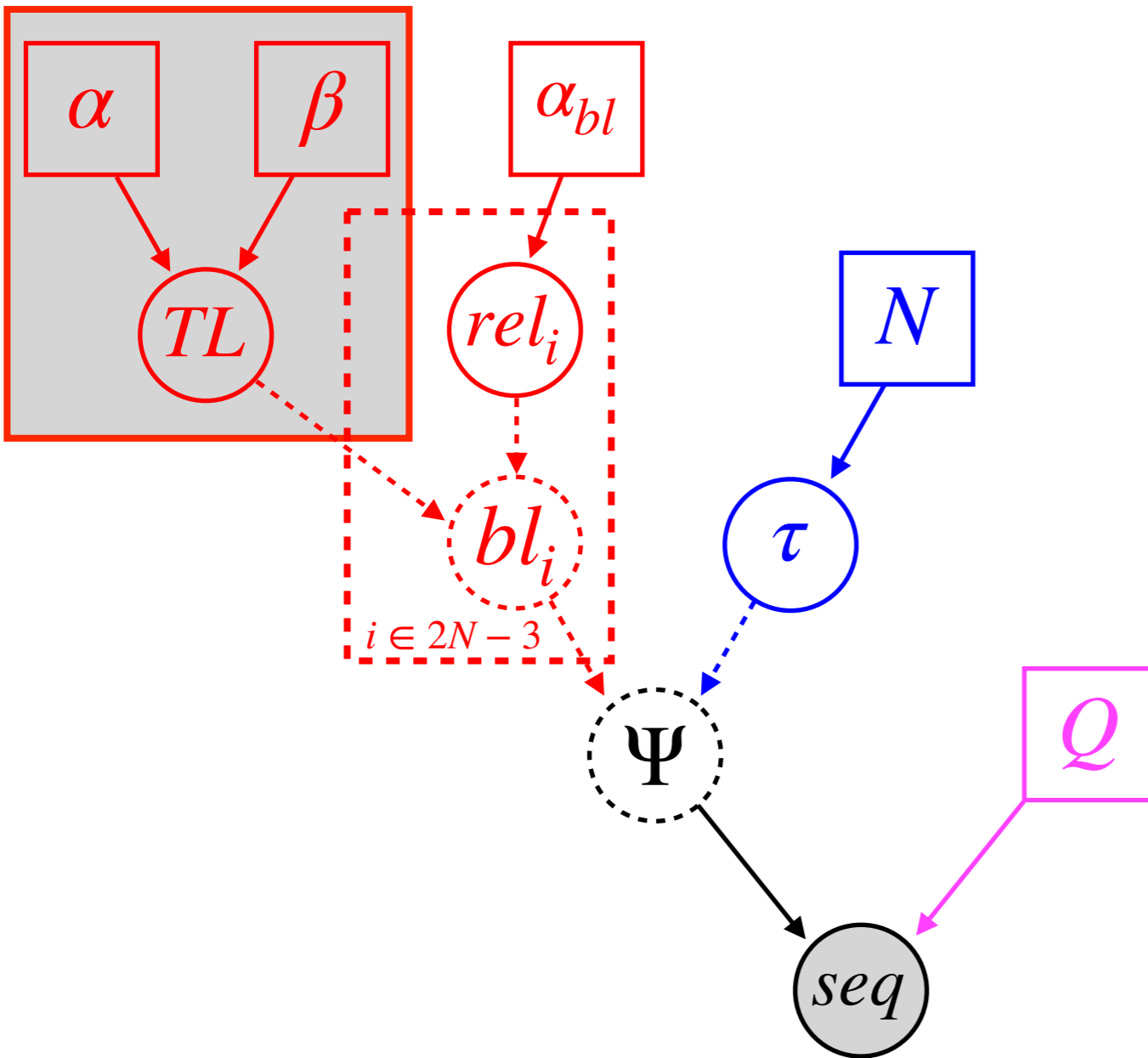
```
br_lens := rel_branch_lengths * TL
```

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Q <- fnJC(4)
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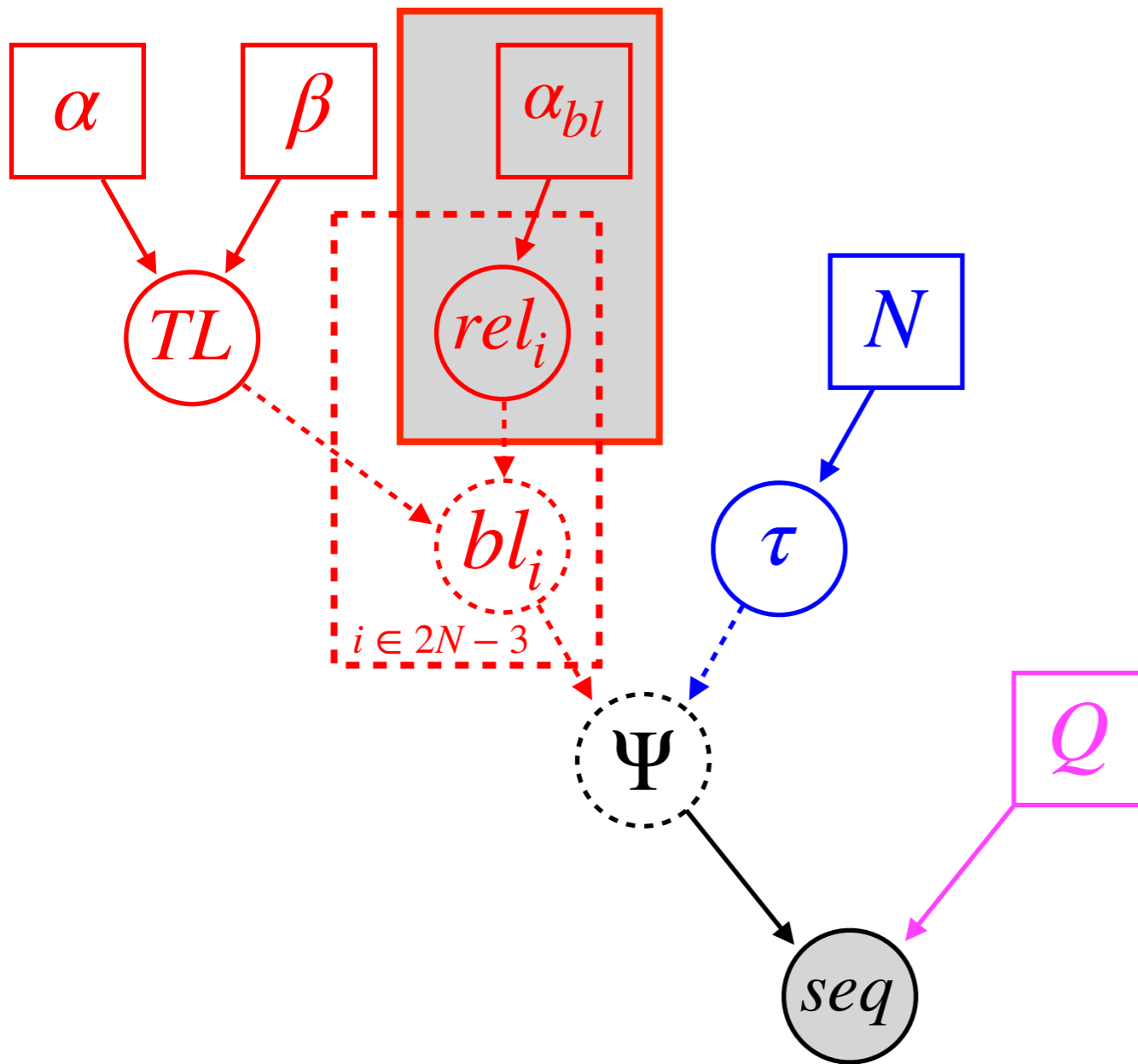
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psi := treeAssembly(topology, br_lens)
```

```
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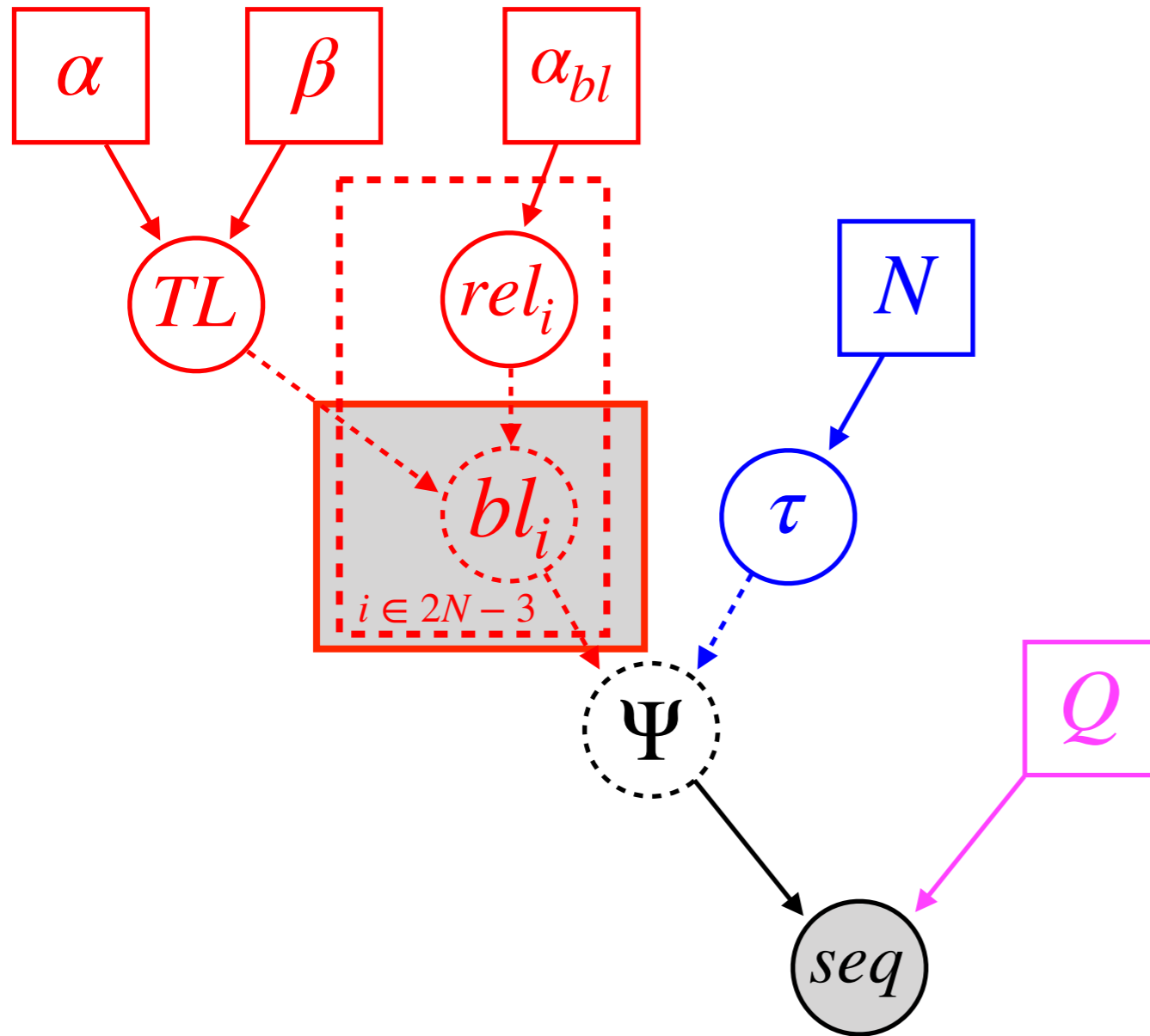
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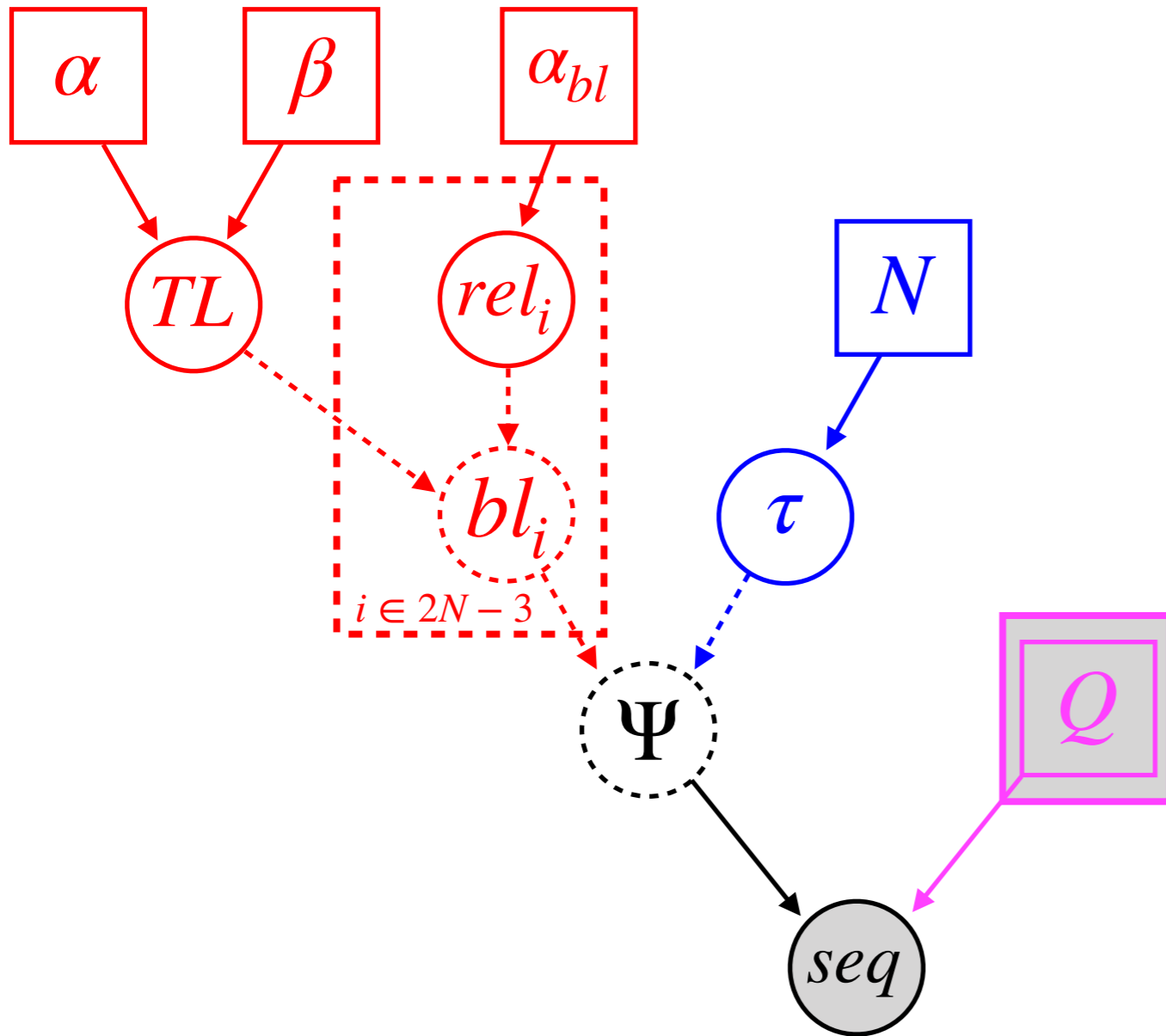
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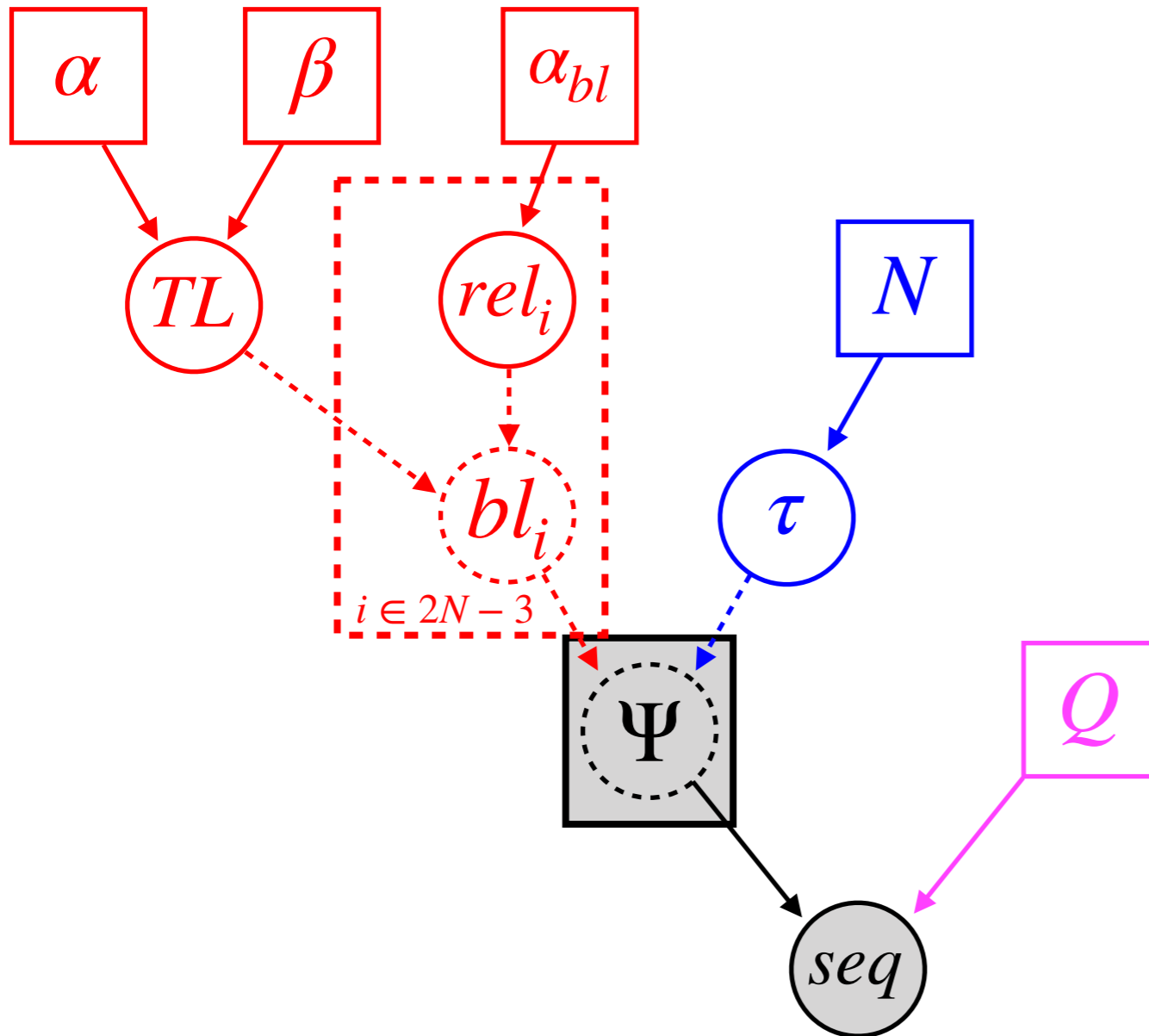
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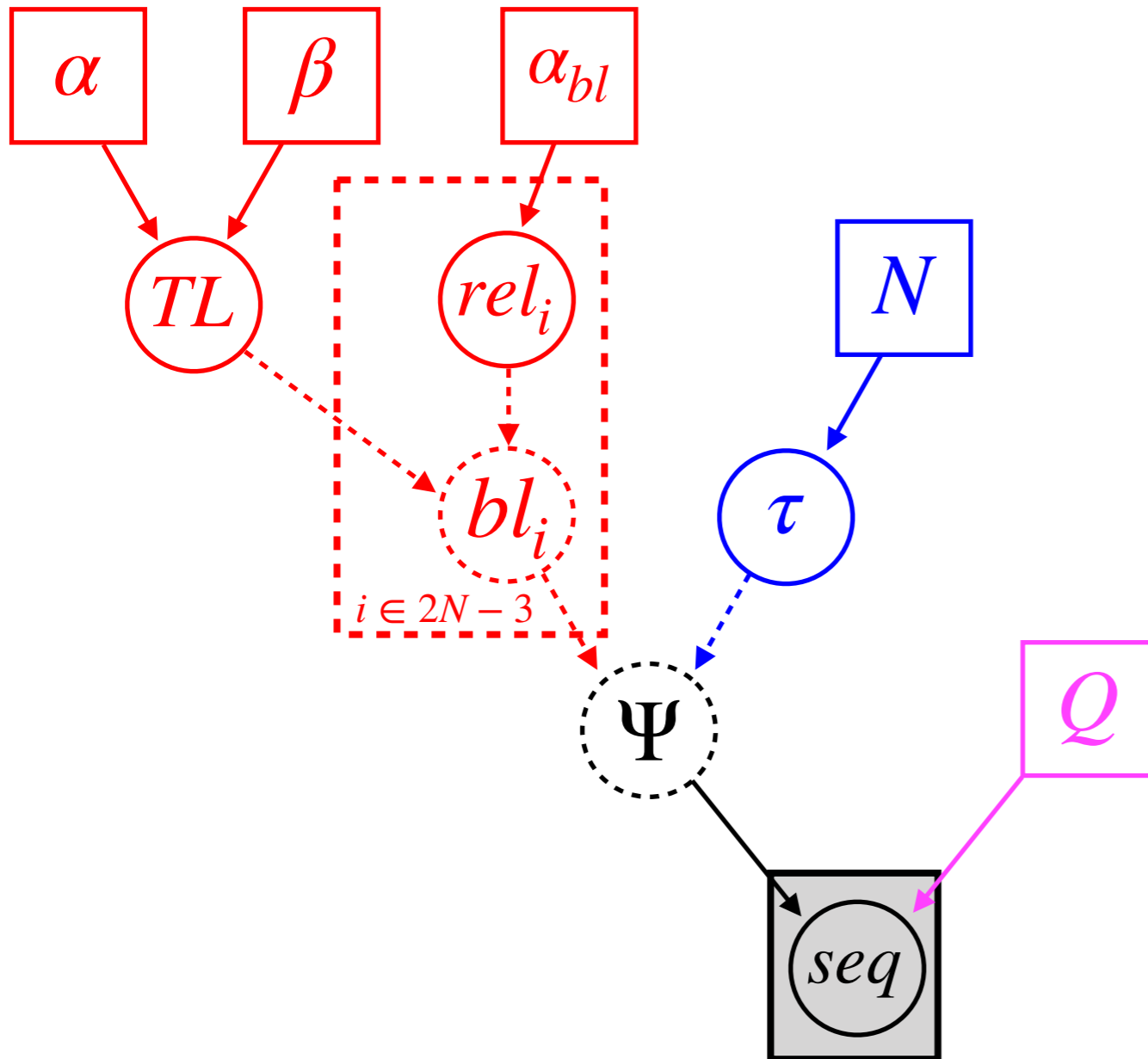
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Jukes-Cantor



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Jukes-Cantor

TL

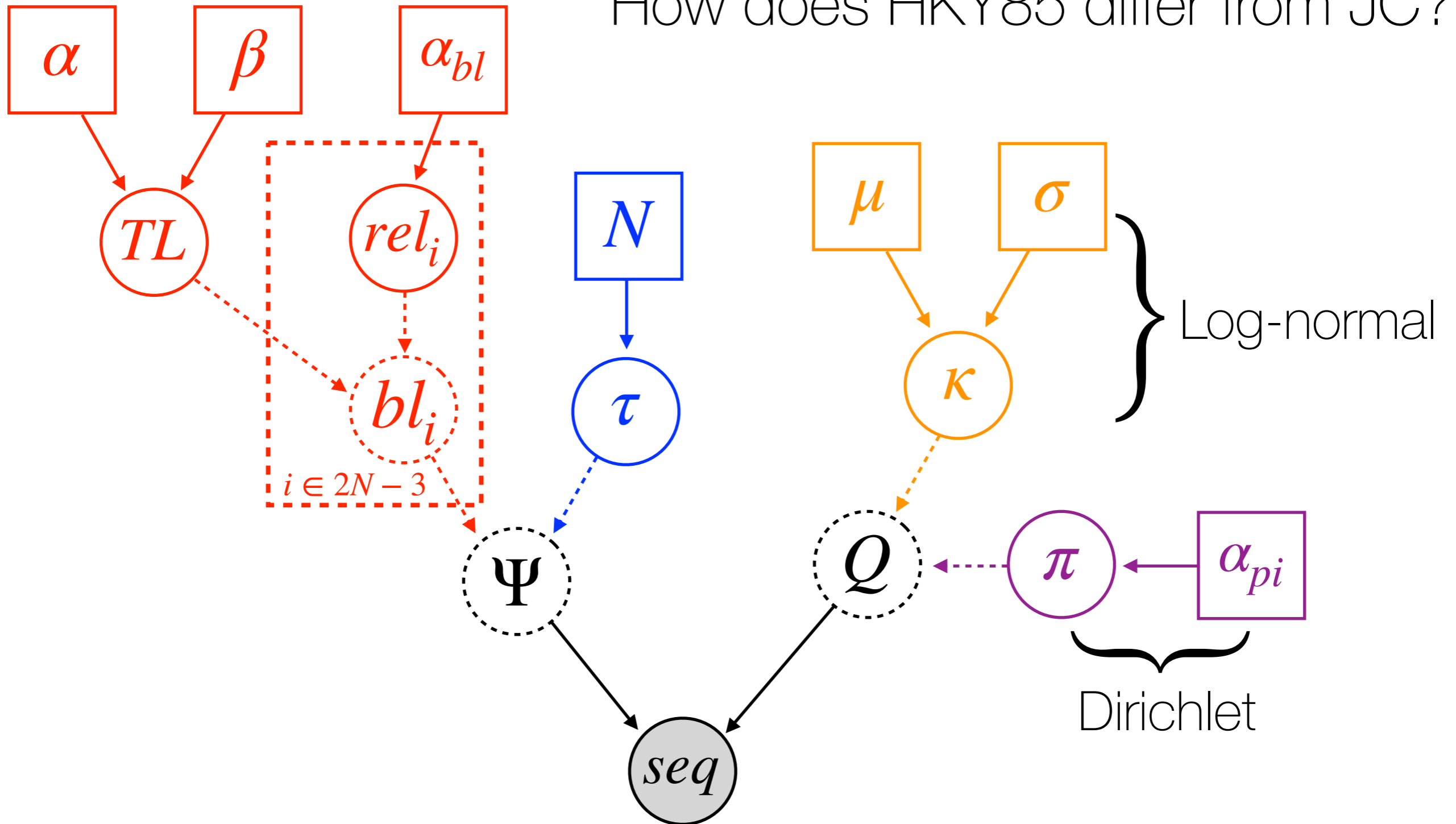
rel_i

τ

Inferring these parameters
under Jukes-Cantor.

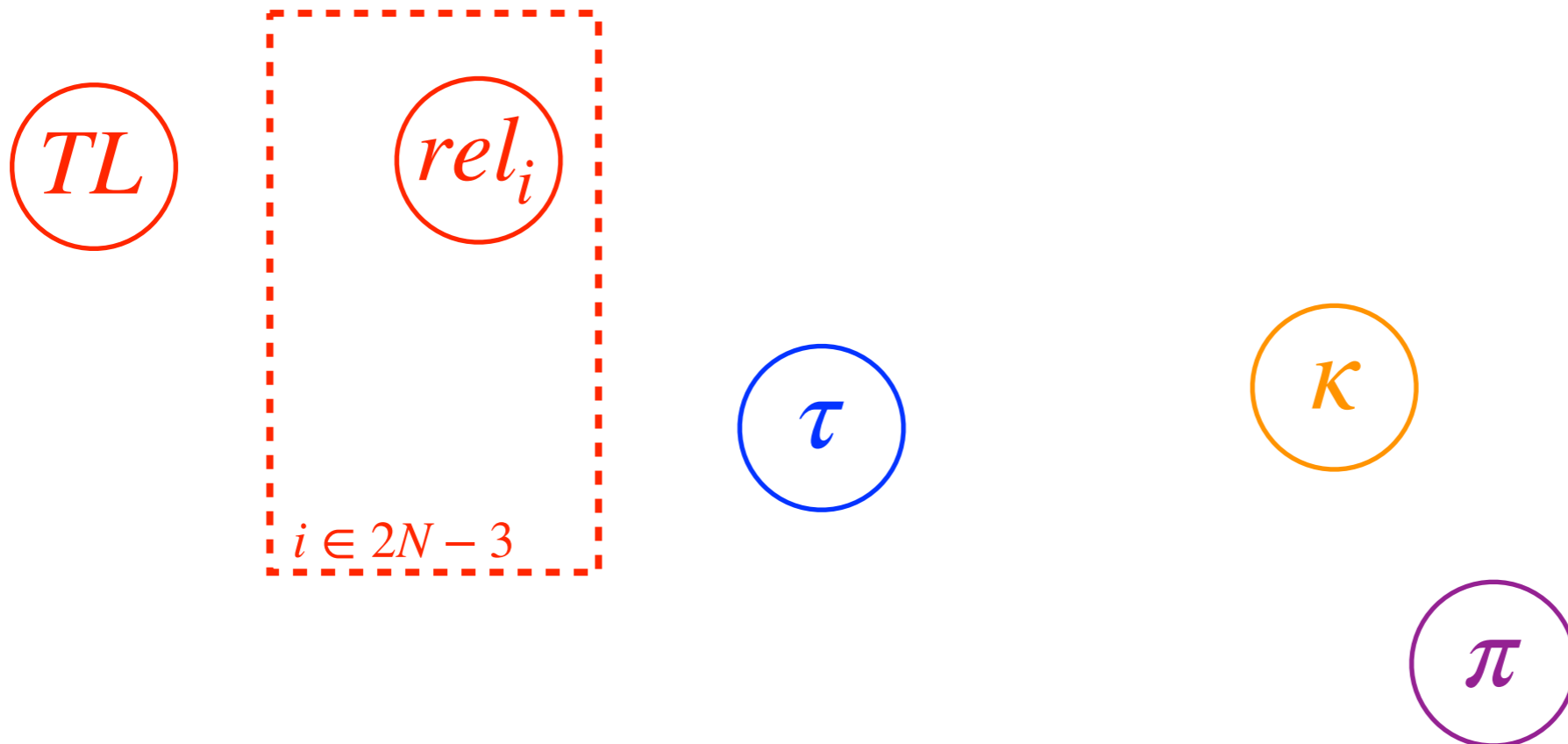
HKY85

How does HKY85 differ from JC?



HKY85

How does HKY85 differ from JC?



HKY85

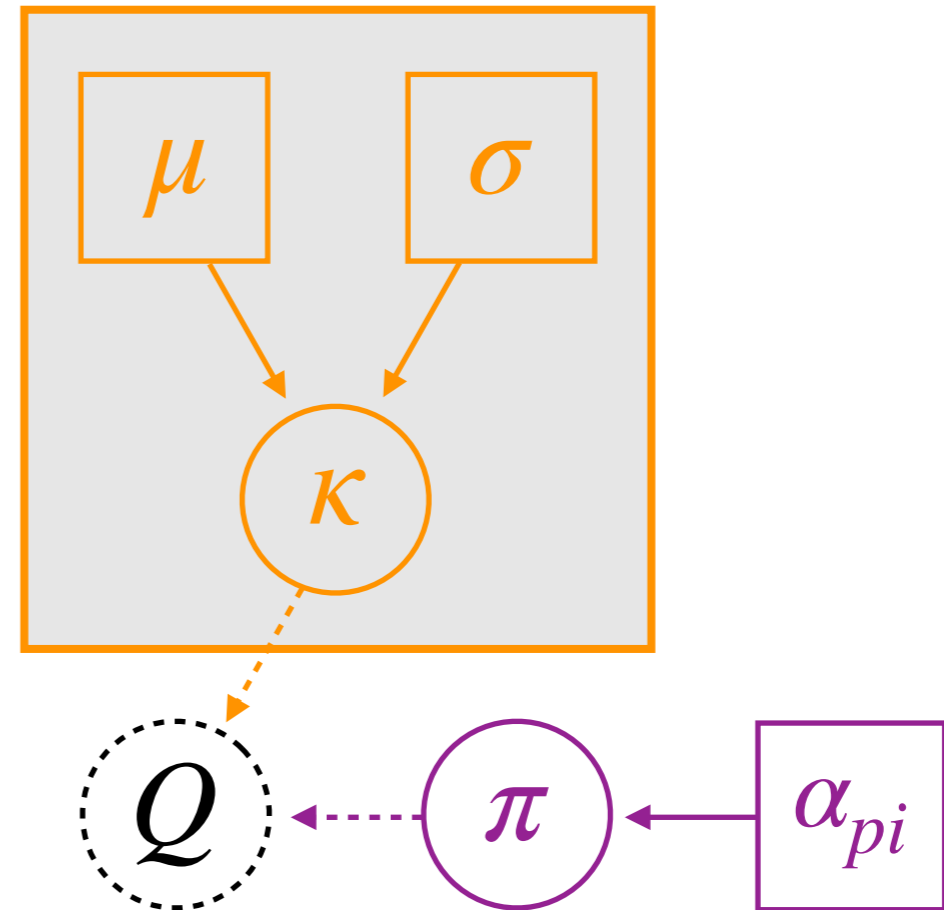
How does HKY85 differ from JC?

Transition-Transversion Rate Ratio

```
mu <- 0  
sigma <- 1  
kappa ~ dnLognormal(mu,sigma)
```

```
alpha_pi <- 1.0  
pi ~ dnDirichlet( rep(alpha_pi,4) )
```

```
Q := fnHKY(kappa,pi)
```



HKY85

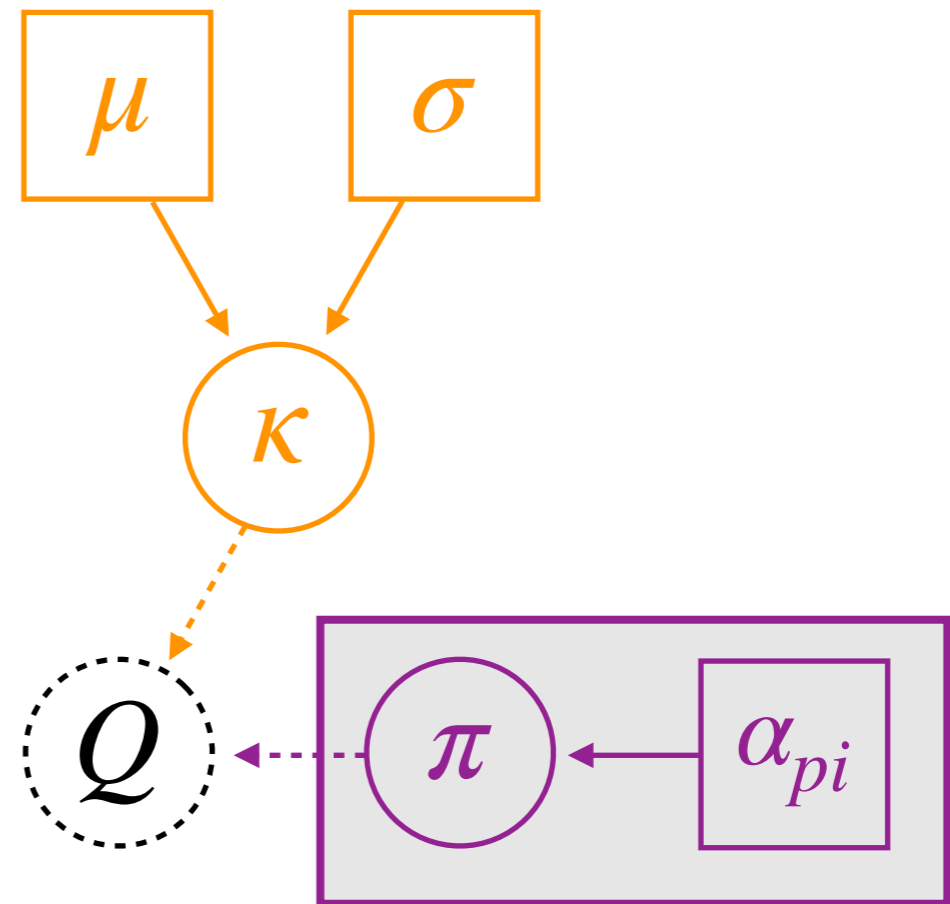
How does HKY85 differ from JC?

```
mu <- 0  
sigma <- 1  
kappa ~ dnLognormal(mu,sigma)
```

```
alpha_pi <- 1.0  
pi ~ dnDirichlet( rep(alpha_pi,4) )
```

Variable Base Frequencies

```
Q := fnHKY(kappa,pi)
```



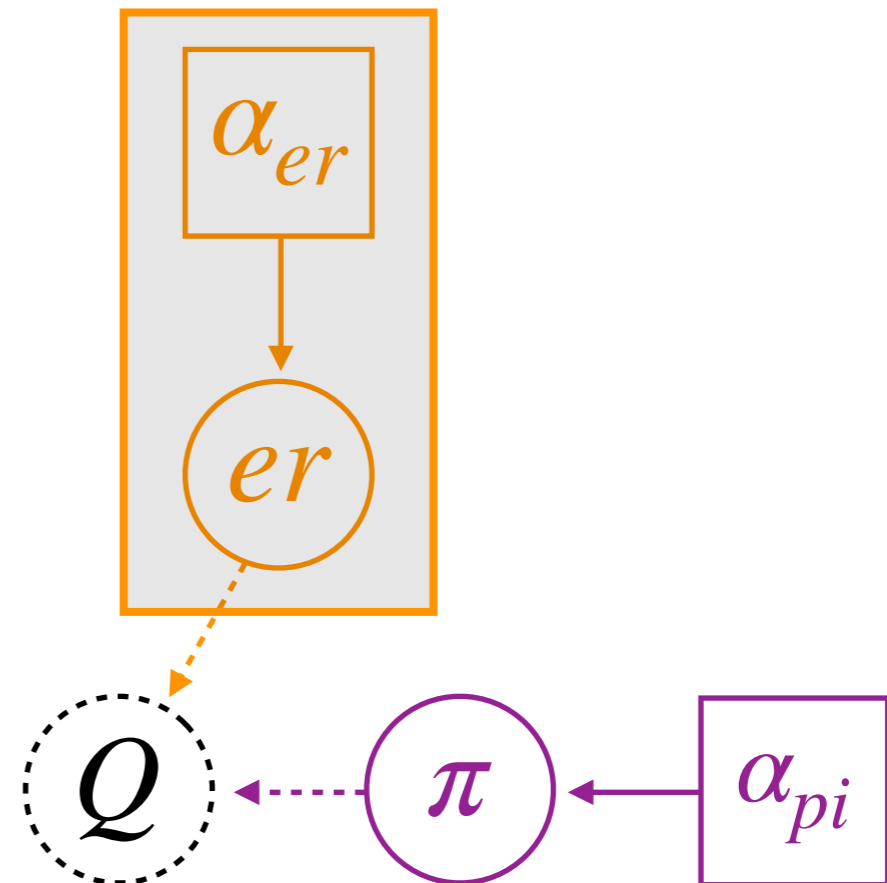
General Time Reversible (GTR)

Variable Exchangeabilities

```
alpha_er <- 1.0  
er ~ dnDirichlet( rep(alpha_er,6) )
```

```
alpha_pi <- 1.0  
pi ~ dnDirichlet( rep(alpha_pi,4) )
```

```
Q := fnGTR(er,pi)
```



Things to Remember

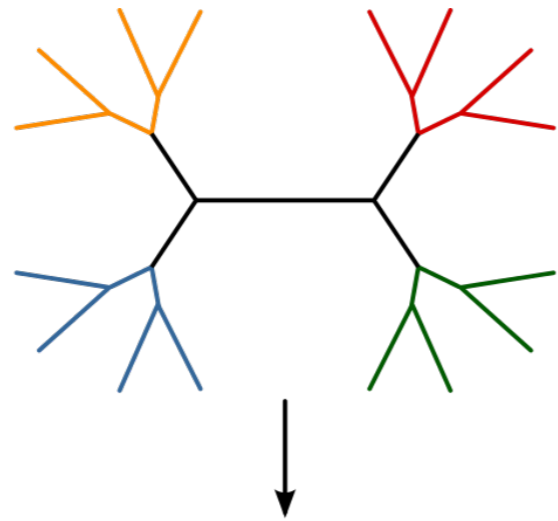
As we add more stochastic nodes, **remember to assign moves** to all of them!

Vectors modeled with a Dirichlet should have a joint proposal mechanism, since they need to sum to 1. Here are a couple:

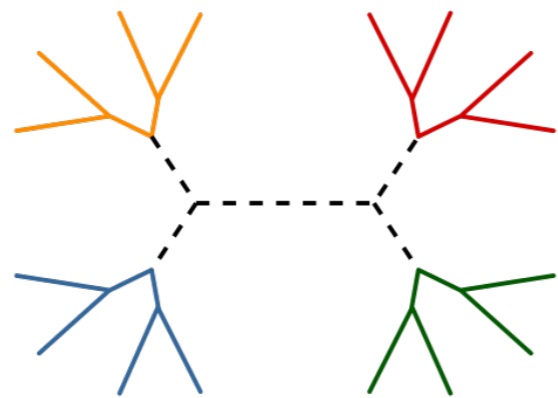
`mvBetaSimplex`
`mvDirichletSimplex`

You'll often want to run analyses on the same dataset with multiple models. Start by saving your simplest model into a text file (.rev). Then, copy and paste the code into a file for the next model and adjust as necessary. Keep going as you work your way to more complex models.

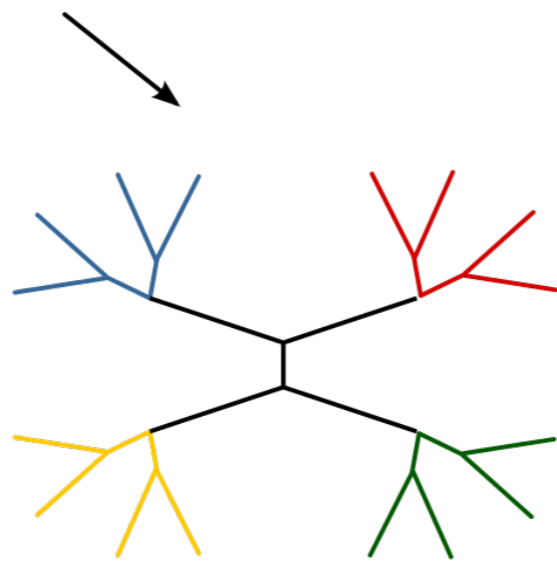
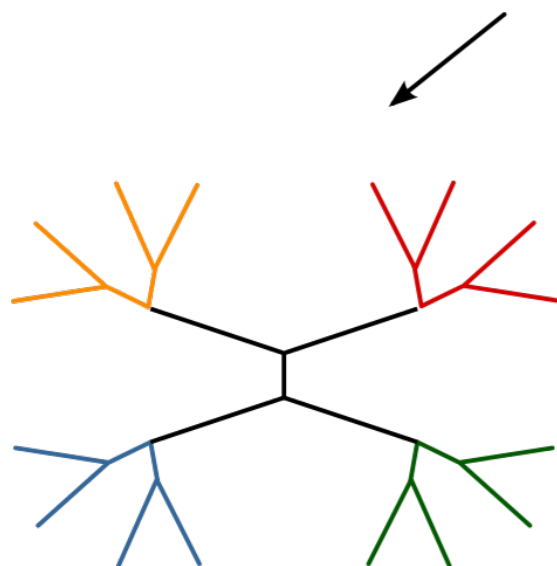
Topology Moves



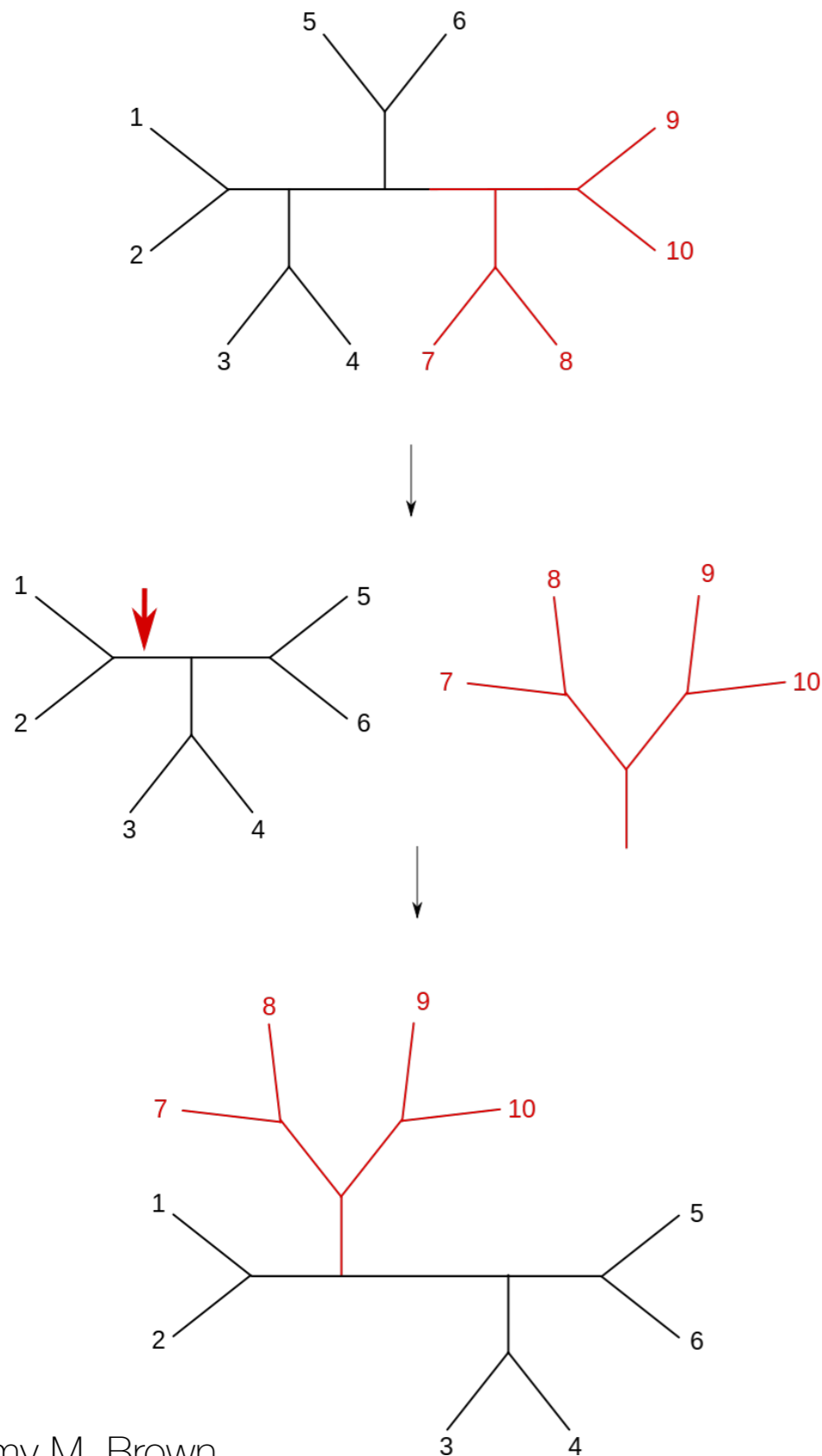
Nearest-Neighbor
Interchange
(NNI)



The most minor
topological move.



Topology Moves

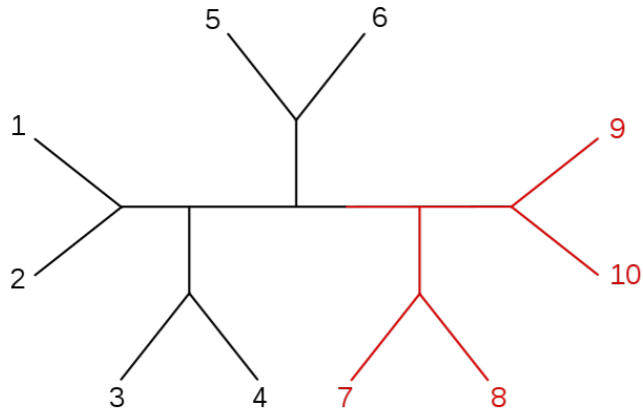


Subtree Prune and Regraft
(SPR)

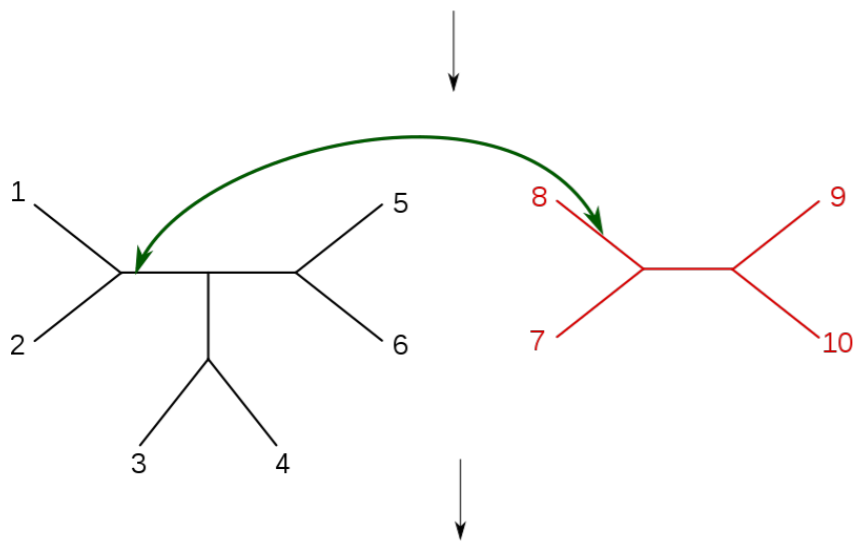
Intermediate topological move.

Often a good balance between exploring new tree space without disrupting things too much.

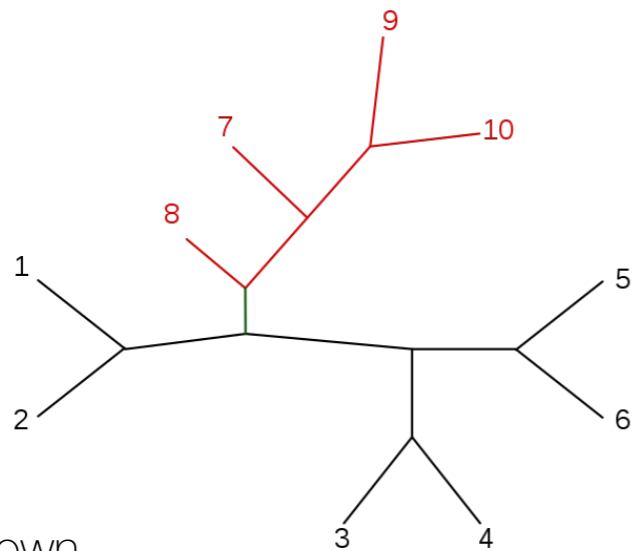
Topology Moves



Tree Bisection and Reconnection
(TBR)

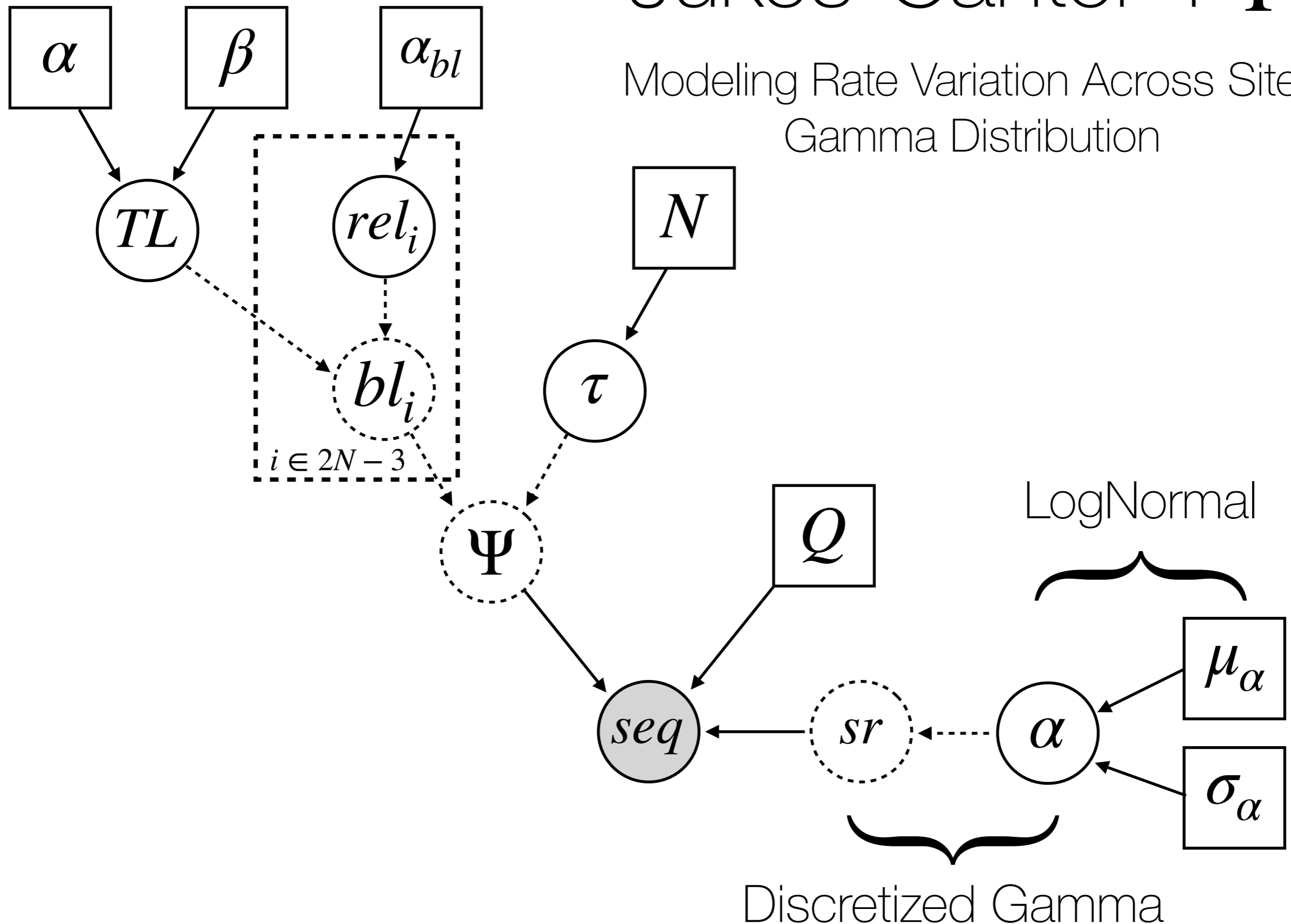


Disruptive topological move.



Jukes-Cantor + Γ

Modeling Rate Variation Across Sites
Gamma Distribution



Jukes-Cantor + Γ

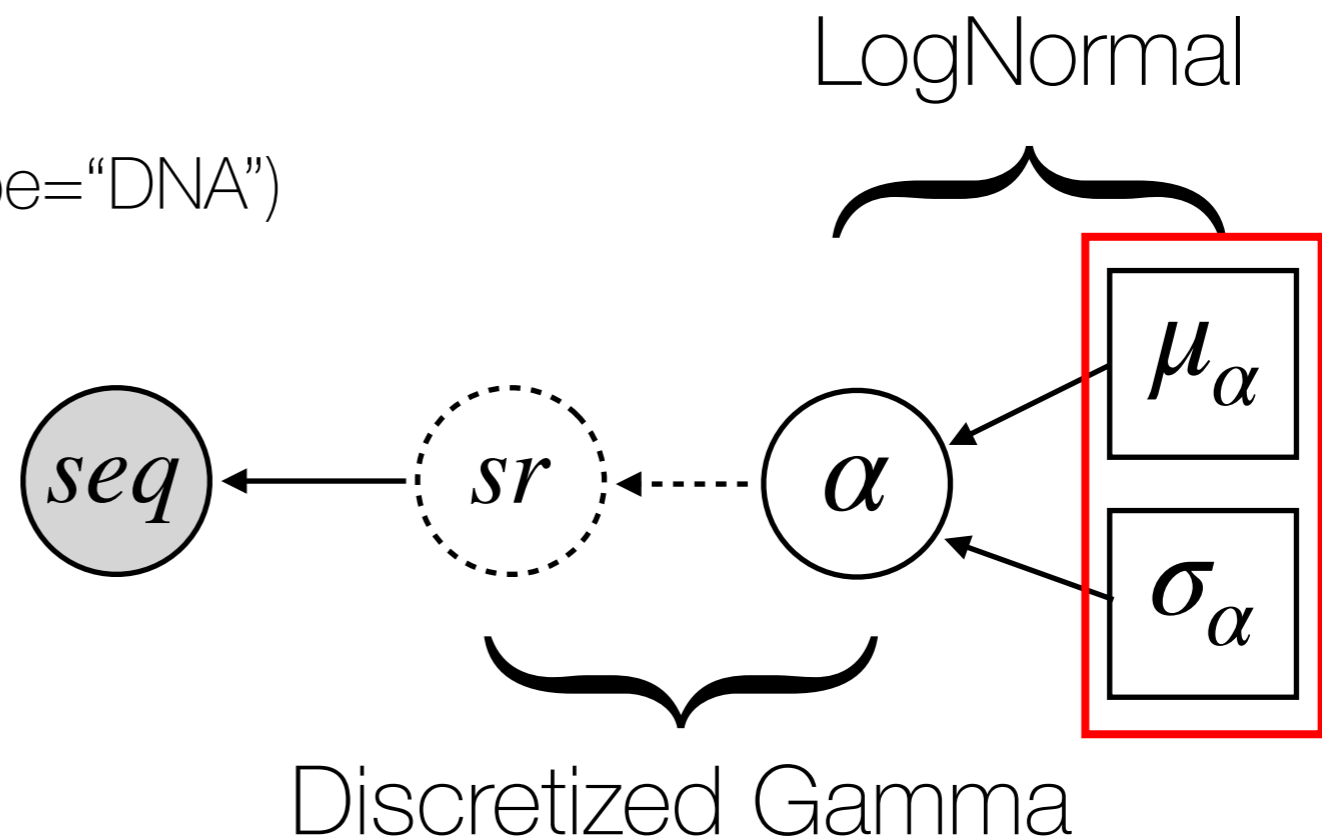
Modeling Rate Variation Across Sites
Gamma Distribution

```
mu_a <- ln(5.0)  
sigma_a <- 0.587405
```

```
alpha ~ dnLognormal( mu_a, sigma_a )
```

```
sr := fnDiscretizeGamma( alpha, alpha, 4, false )
```

```
seq ~ dnPhyloCTMC(tree=psi, Q=Q,  
siteRates=sr, type="DNA")
```



Jukes-Cantor + Γ

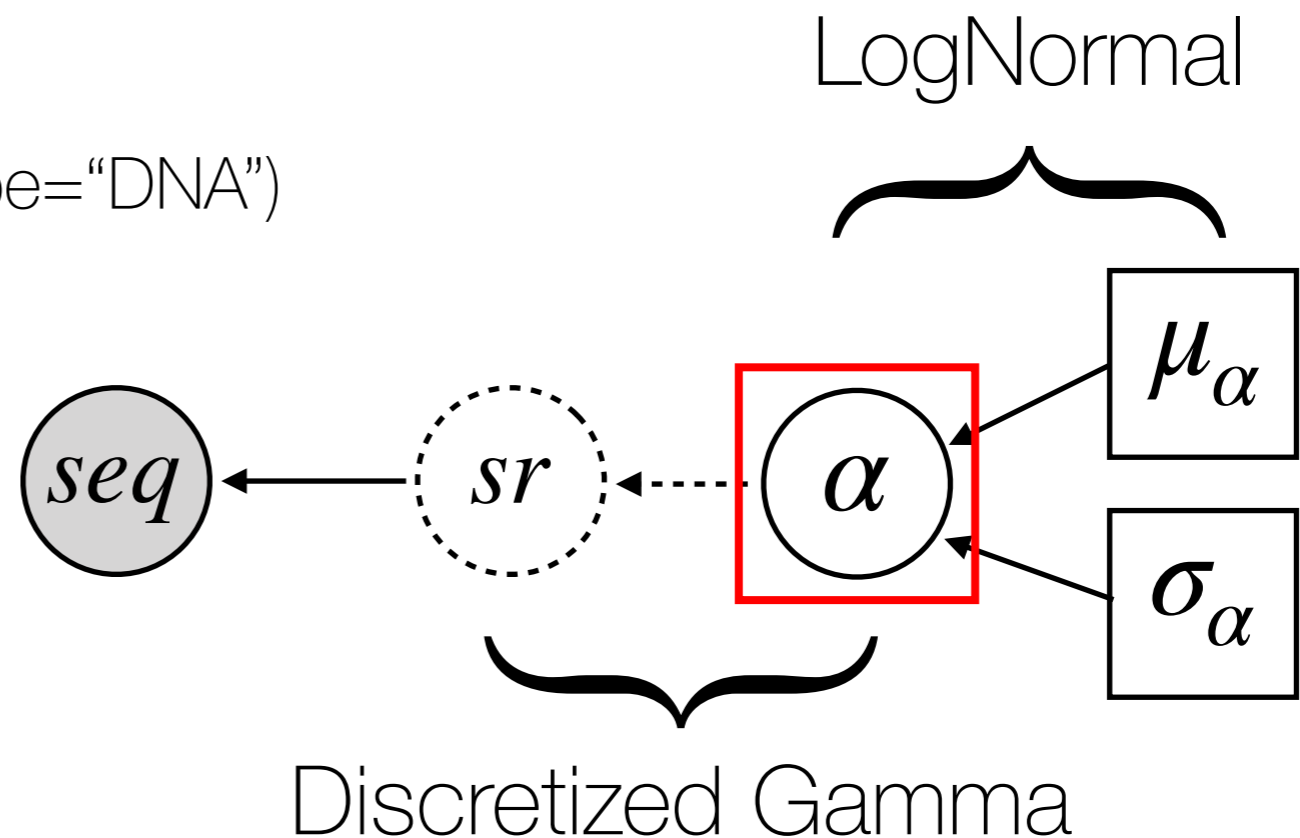
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Jukes-Cantor + Γ

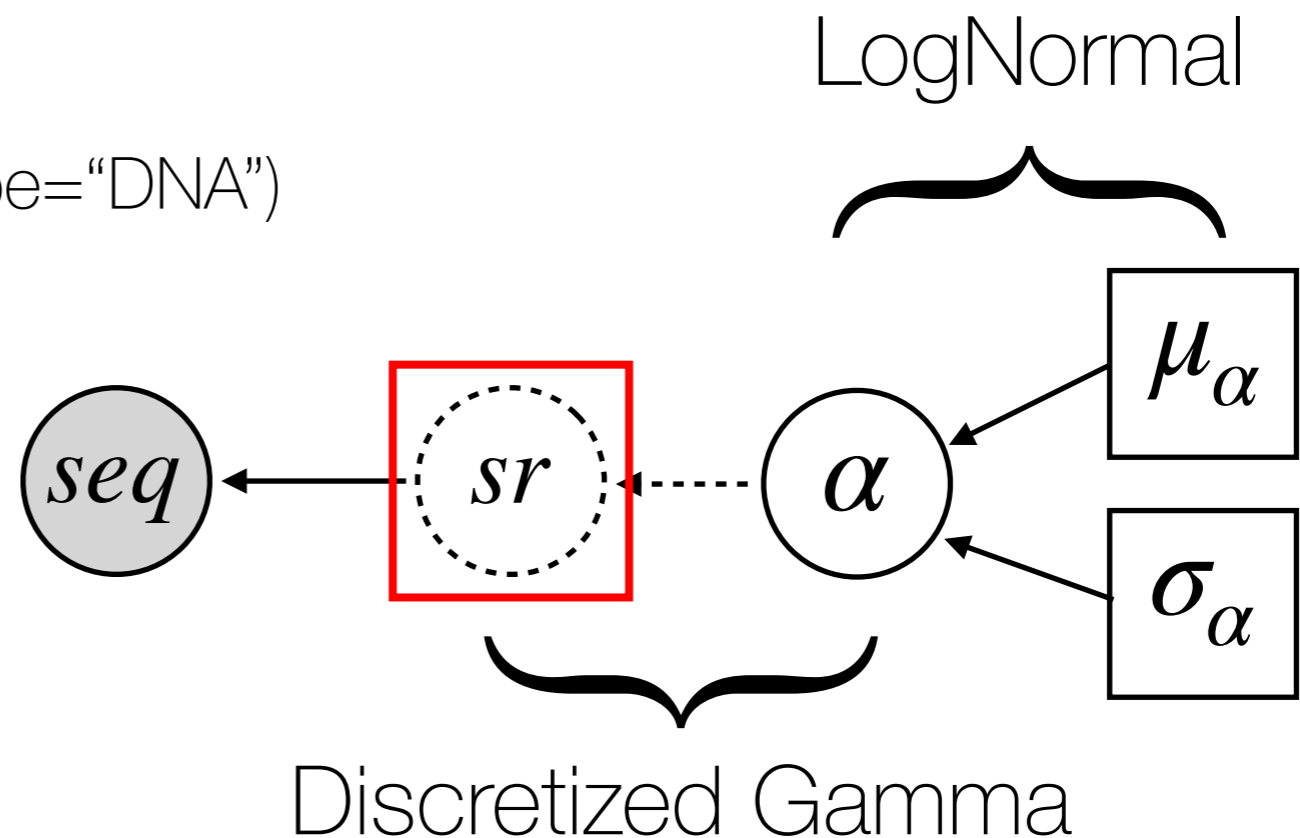
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```



Jukes-Cantor + Γ

Modeling Rate Variation Across Sites
Gamma Distribution

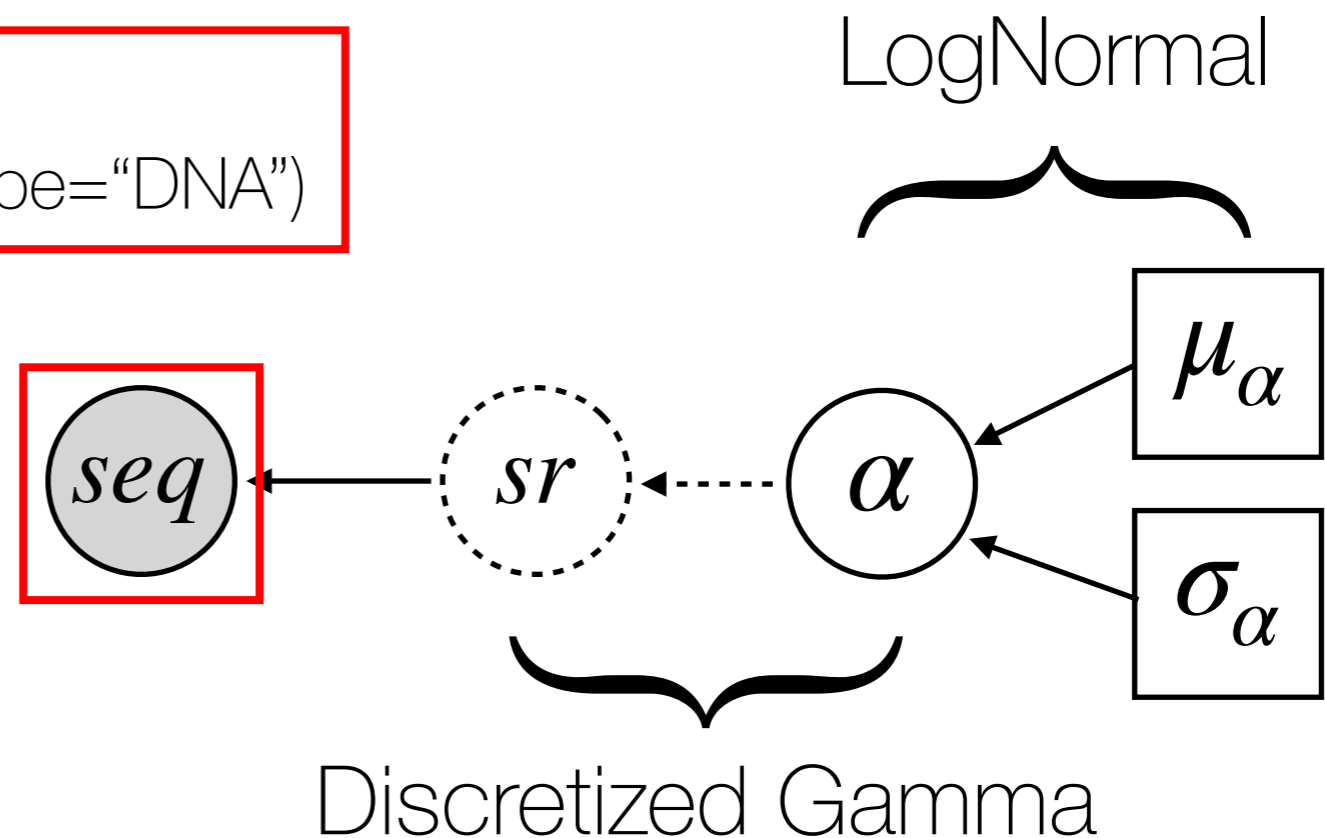
```
mu_a <- ln(5.0)
```

```
sigma_a <- 0.587405
```

```
alpha ~ dnLognormal( mu_a, sigma_a )
```

```
sr := fnDiscretizeGamma( alpha, alpha, 4, false )
```

```
seq ~ dnPhyloCTMC(tree=psi,Q=Q,  
siteRates=sr,type="DNA")
```



Jukes-Cantor + Γ

Modeling Rate Variation Across Sites
Gamma Distribution

```
mu_a <- ln(5.0)
```

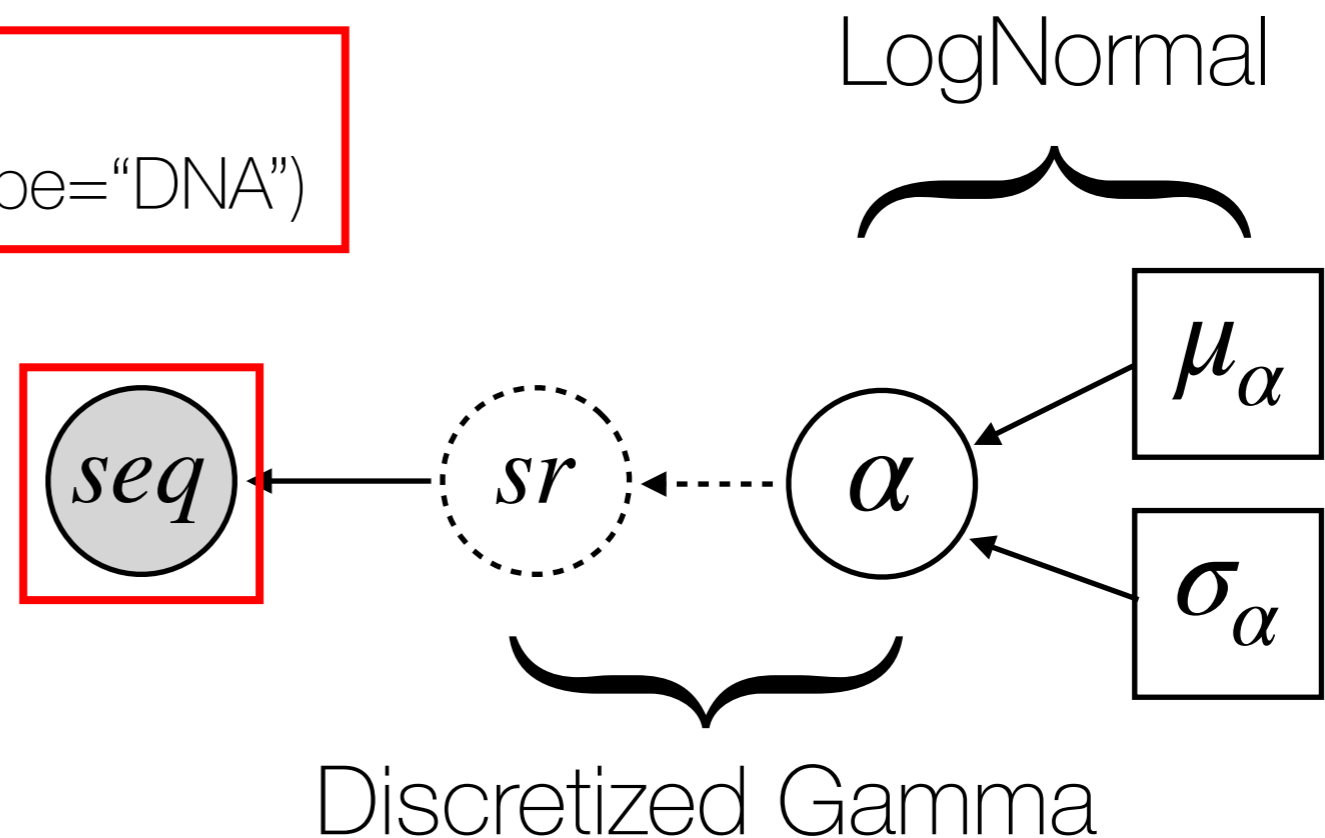
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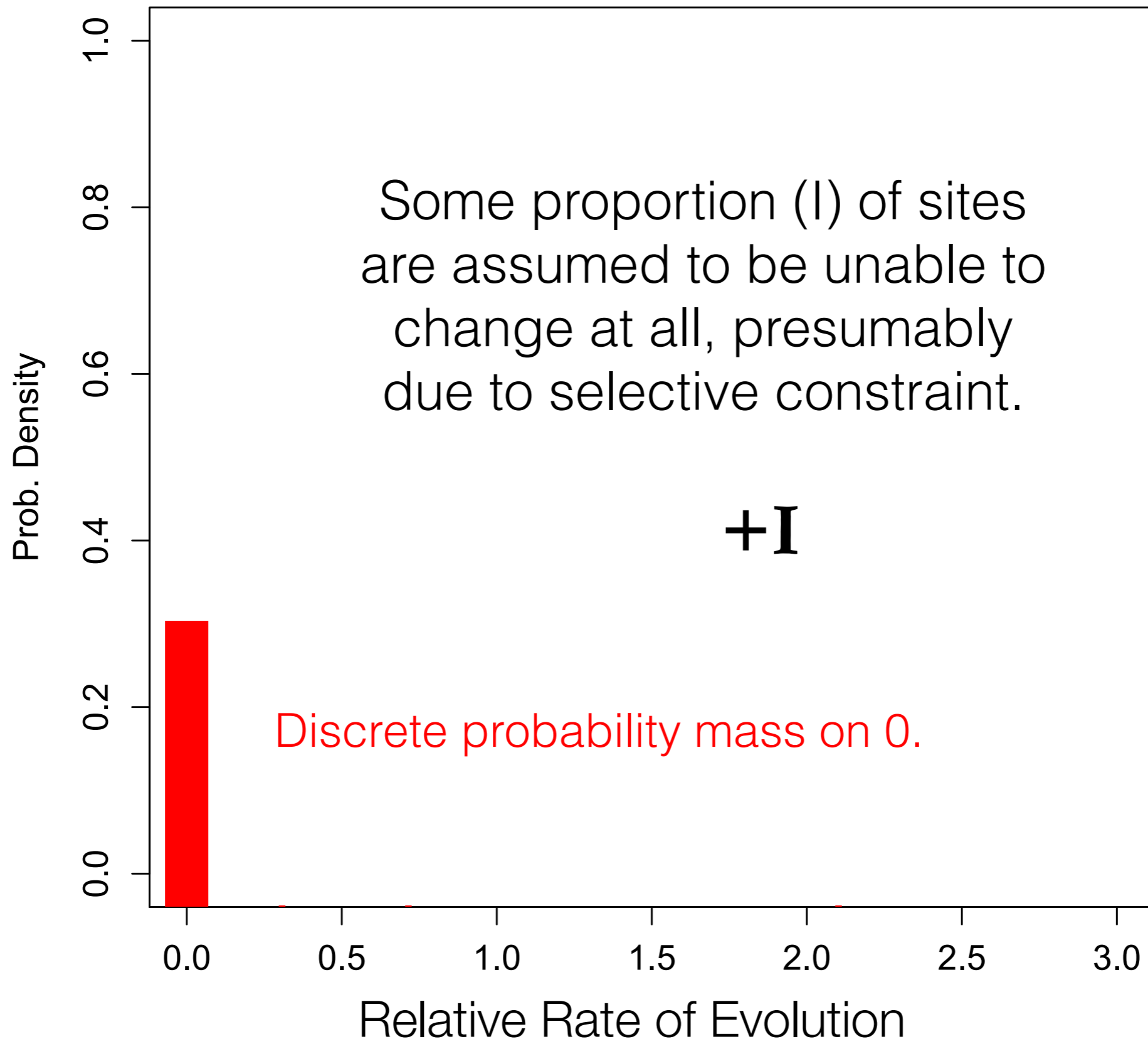
```
seq ~ dnPhyloCTMC(tree=psi,Q=Q,  
siteRates=sr,type="DNA")
```

Can add + Γ to any
“standard” model of
sequence evolution.



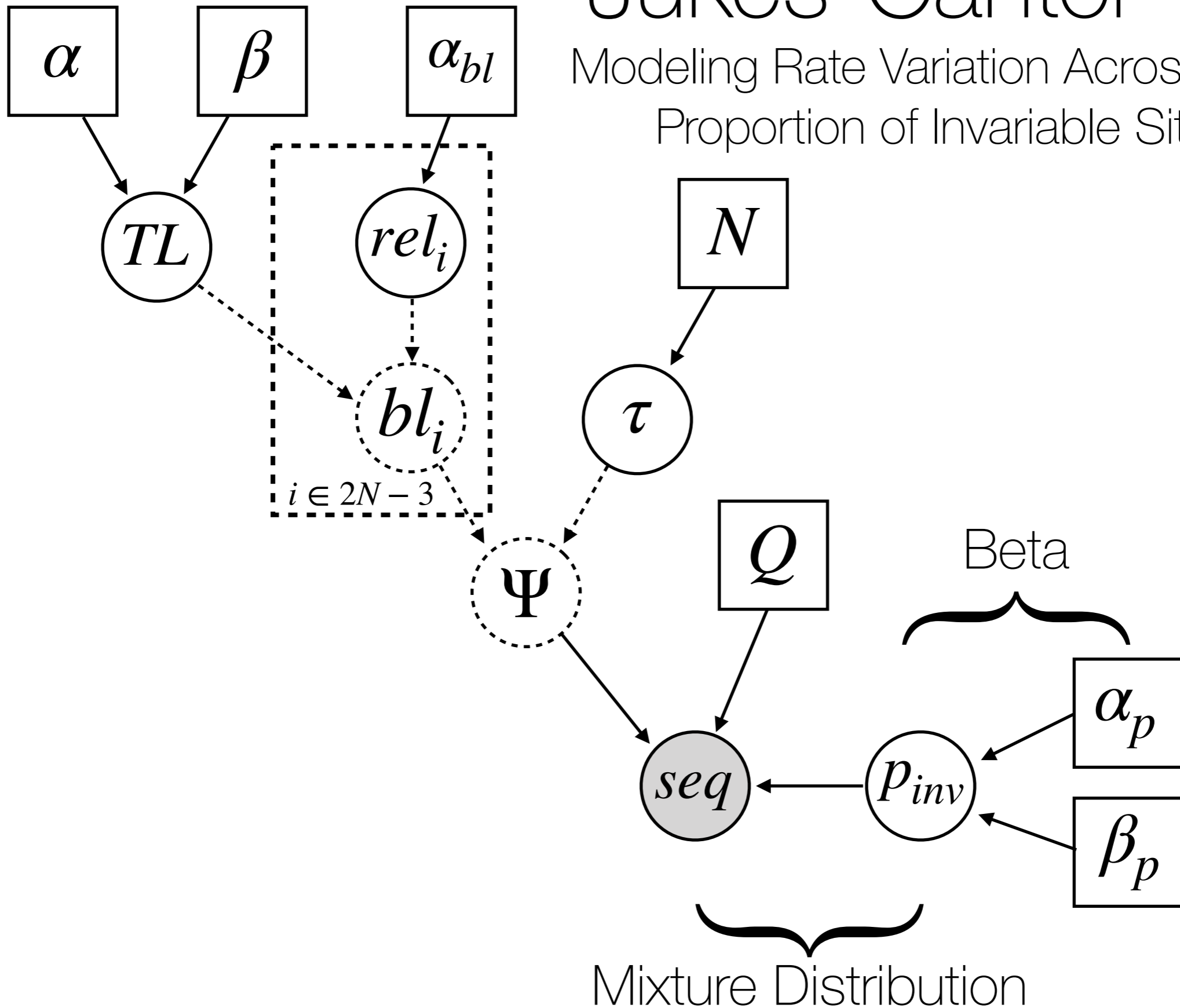
Modeling Rate Variation Across Sites

Proportion of Invariable Sites



Jukes-Cantor + I

Modeling Rate Variation Across Sites
Proportion of Invariable Sites



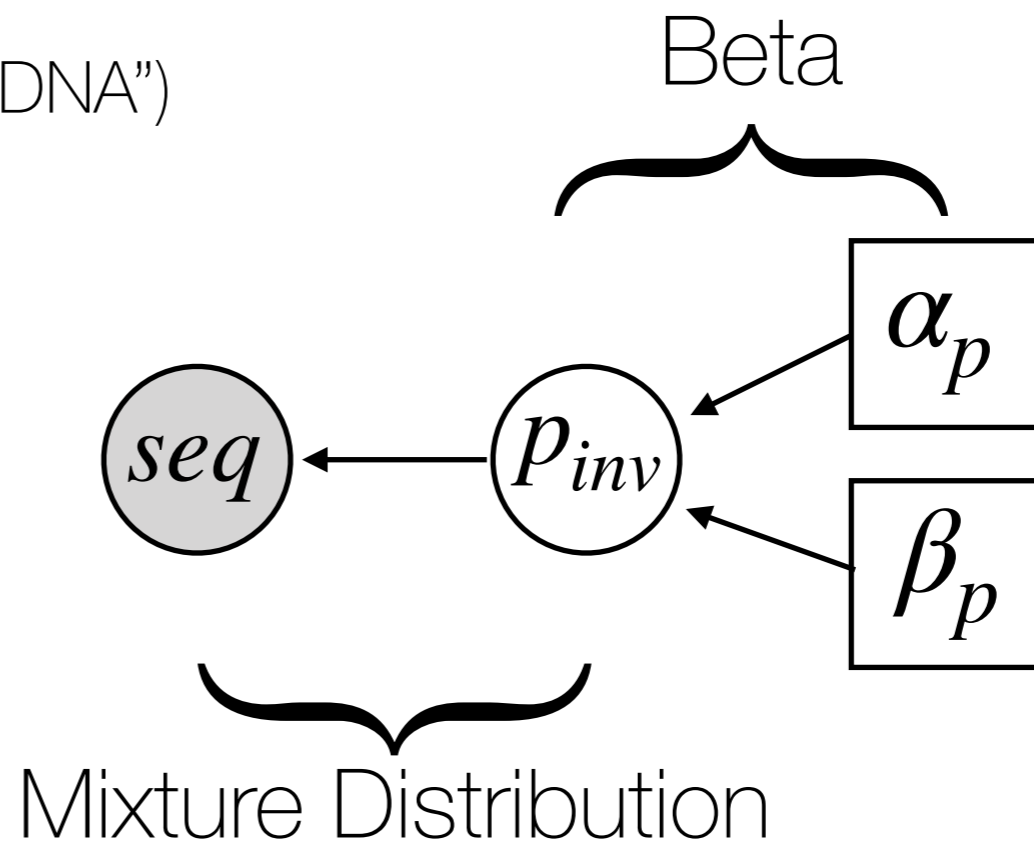
Jukes-Cantor + I

Modeling Rate Variation Across Sites
Proportion of Invariable Sites

```
alpha_p <- 1.0  
beta_p <- 1.0
```

```
p_inv ~ dnBeta( alpha_p, beta_p )
```

```
seq ~ dnPhyloCTMC(tree=psi, Q=Q,  
                 pInv=p_inv, type="DNA")
```



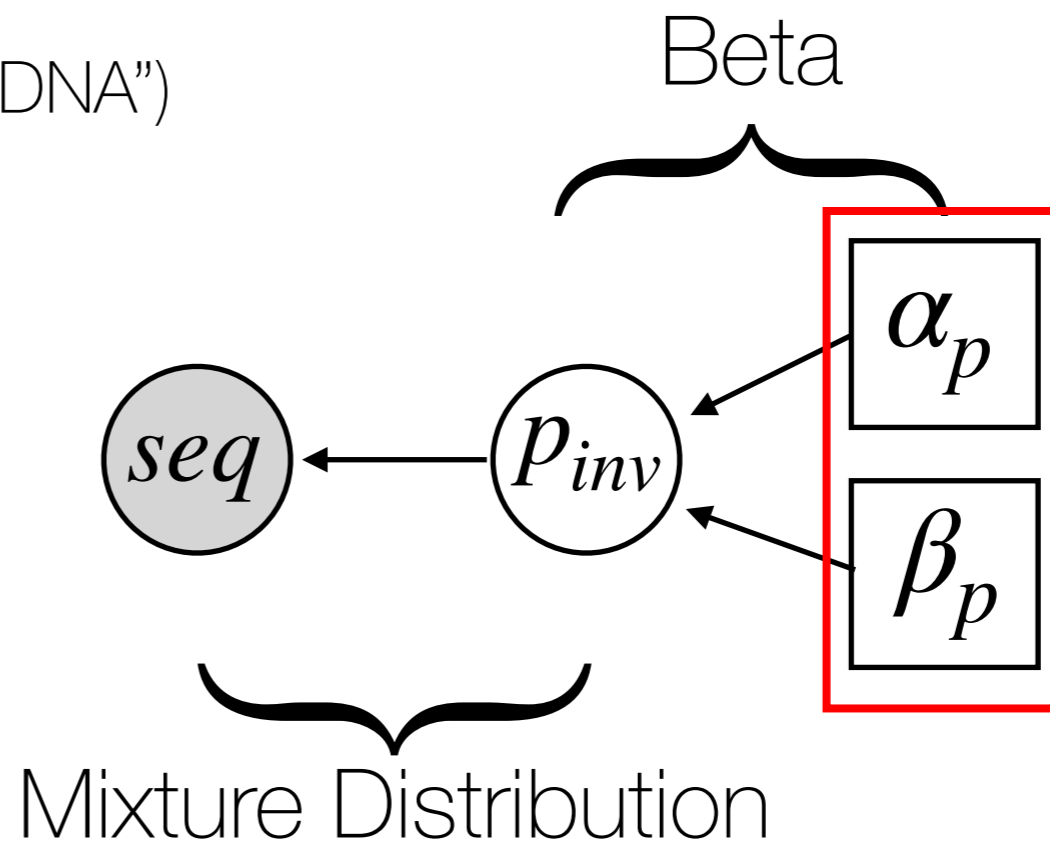
Jukes-Cantor + I

Modeling Rate Variation Across Sites
Proportion of Invariable Sites

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alpha_p <- 1.0  
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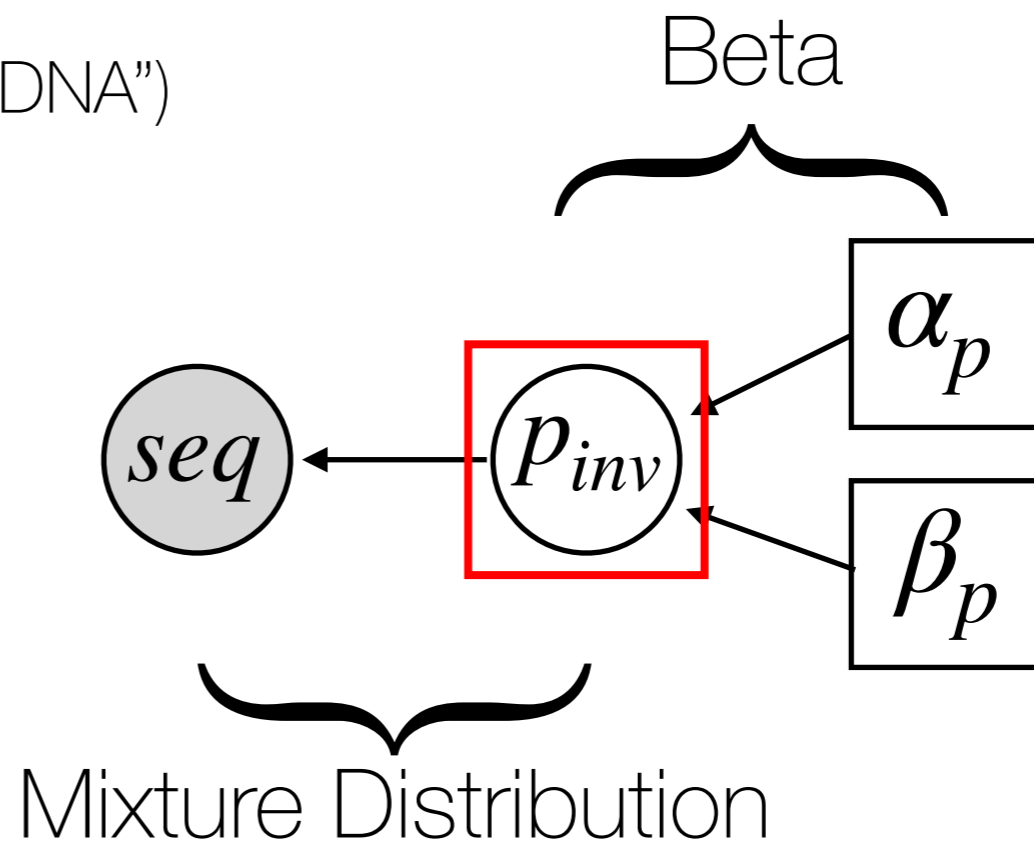
Jukes-Cantor + I

Modeling Rate Variation Across Sites
Proportion of Invariable Sites

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alpha_p <- 1.0  
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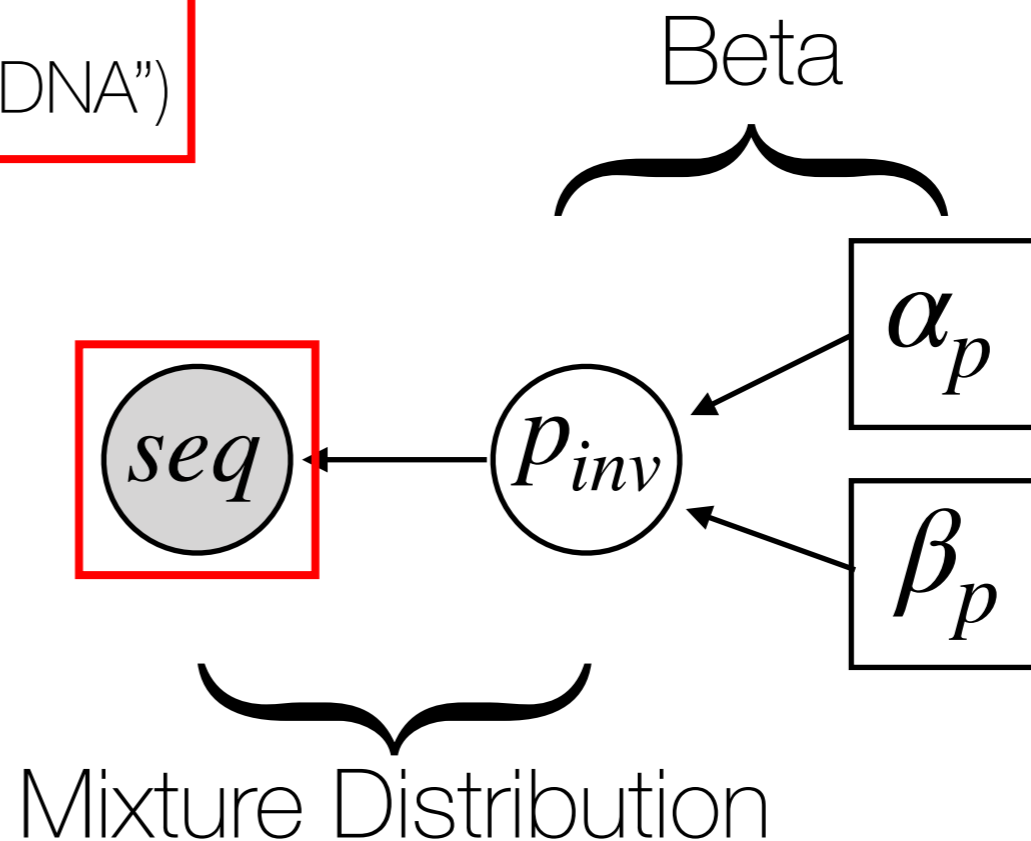
Jukes-Cantor + I

Modeling Rate Variation Across Sites
Proportion of Invariable Sites

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alpha_p <- 1.0  
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Jukes-Cantor + I

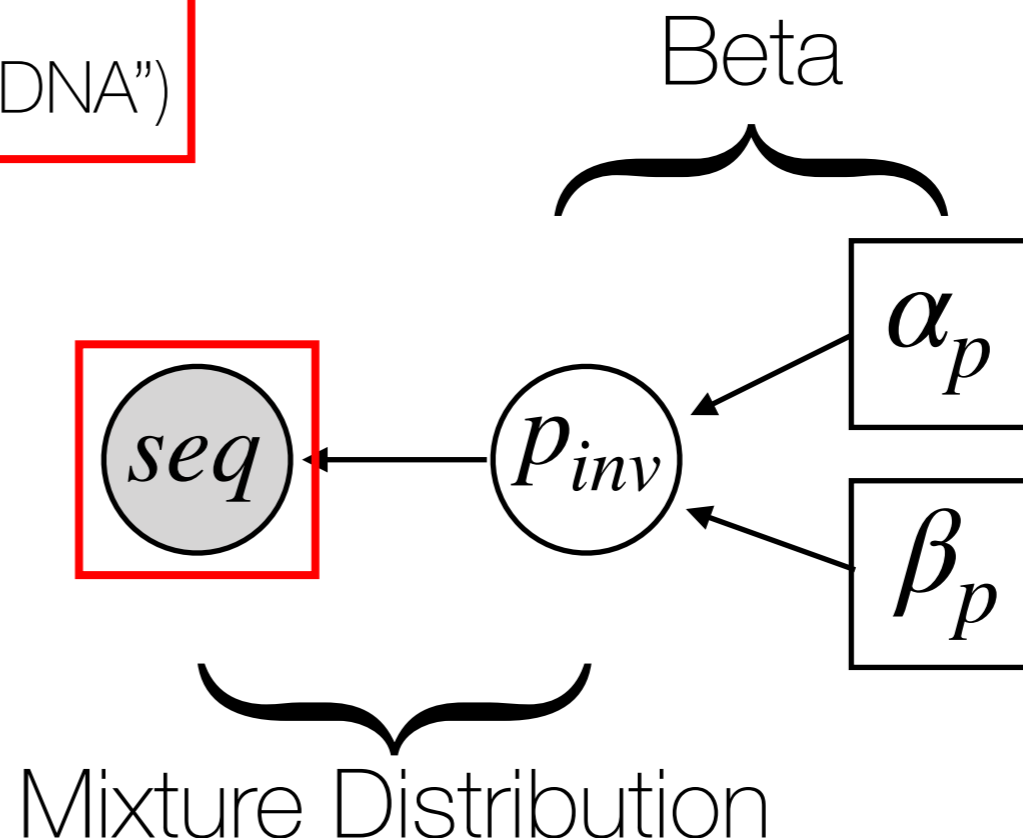
Modeling Rate Variation Across Sites
Proportion of Invariable Sites

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seq ~ dnPhyloCTMC(tree=psi, Q=Q,  
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```

Can also add + I to
any “standard” model
of sequence evolution.



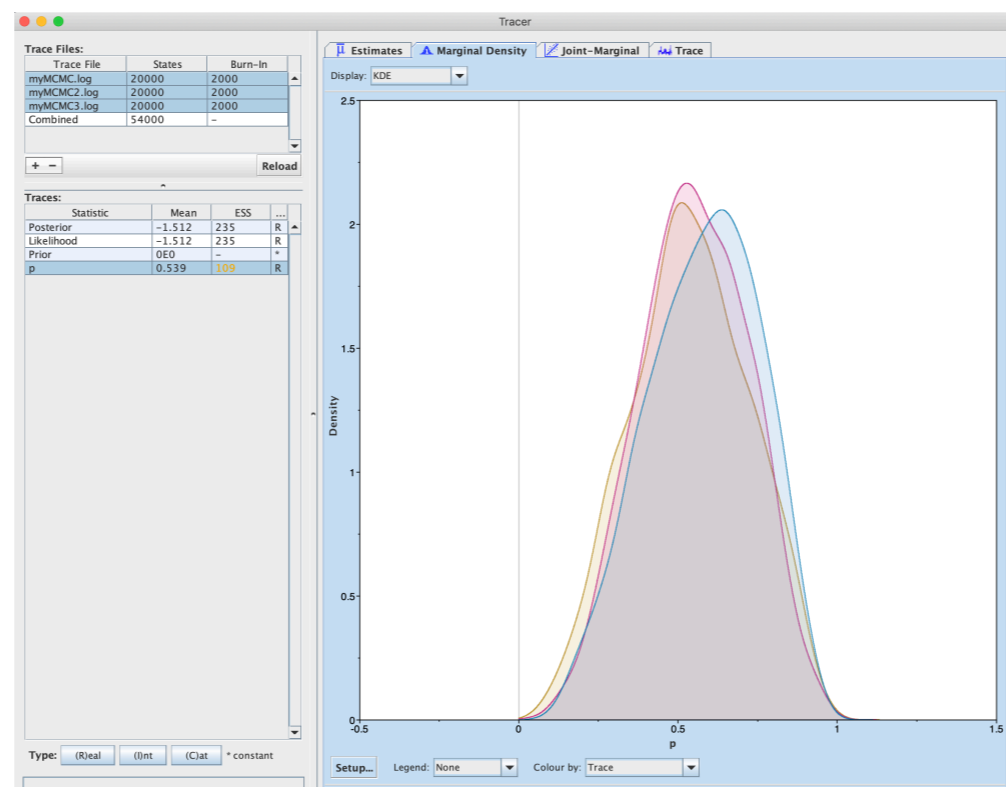
Summarizing Phylogenetic MCMC Output

For scalar (numerical) parameters, we often report:

Mean

Median

95% Highest Posterior Density Interval



Summarizing Phylogenetic MCMC Output

For trees, we often report:

Maximum A Posteriori (MAP) Tree
(“Best” Tree)

```
mapTree(run1Trees)
```

Summarizing Phylogenetic MCMC Output

For trees, we often report:

Maximum A Posteriori (MAP) Tree
(“Best” Tree)

Majority-Rule Consensus Tree
(Formed from all bipartitions with posterior > 0.5)

```
consensusTree(run1Trees,0.5)
```

Summarizing Phylogenetic MCMC Output

For trees, we often report:

Maximum A Posteriori (MAP) Tree
(“Best” Tree)

Majority-Rule Consensus Tree
(Formed from all bipartitions with posterior > 0.5)

Greedy Consensus Tree
(Formed by ranking bipartitions and adding them until tree is fully resolved)

```
consensusTree(run 1 Trees, 0.0)
```

Summarizing Phylogenetic MCMC Output

For trees, we often report:

Maximum A Posteriori (MAP) Tree
(“Best” Tree)

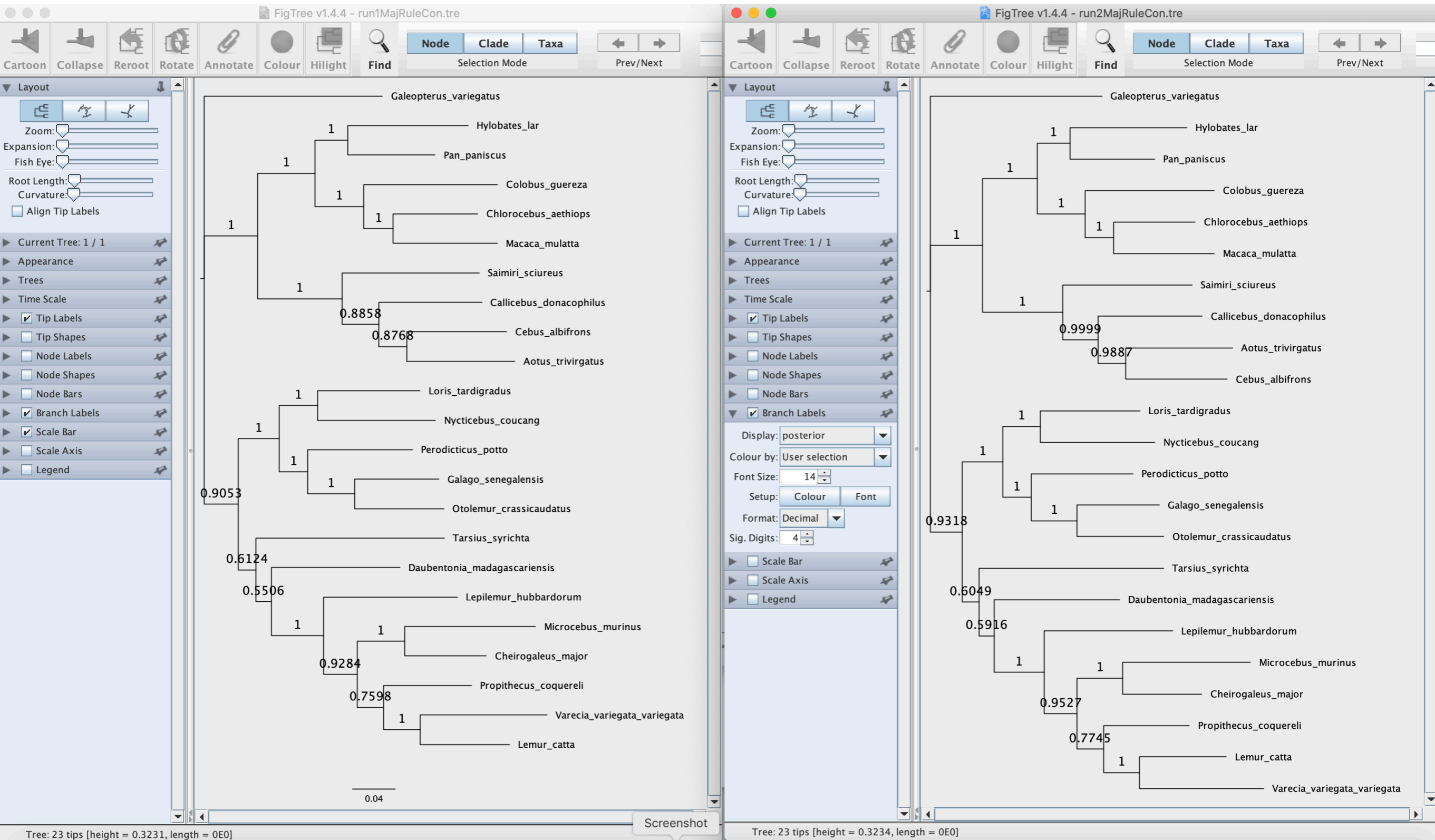
Majority-Rule Consensus Tree
(Formed from all bipartitions with posterior > 0.5)

Greedy Consensus Tree
(Formed by ranking bipartitions and adding them until tree is fully resolved)

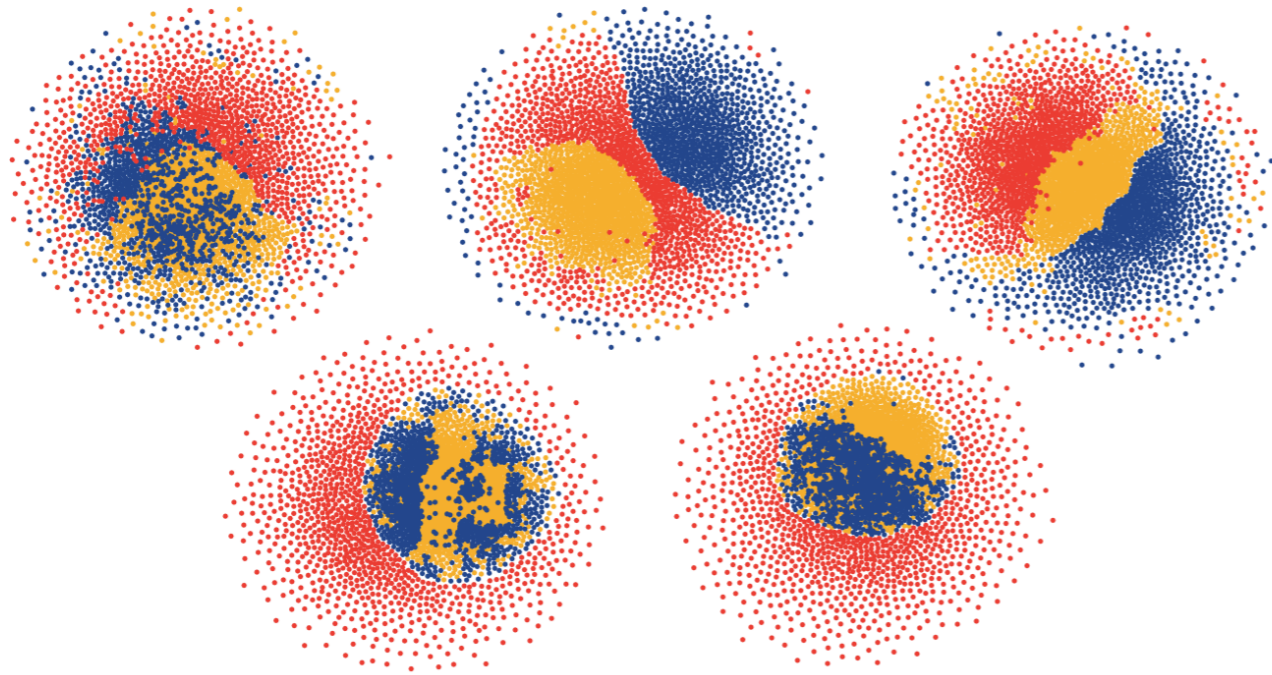
Maximum Clade Credibility Tree
(Sampled tree whose product of bipartition probabilities is greatest)

```
mccTree(run1Trees)
```

Assessing Topological Convergence



Assessing Topological Convergence

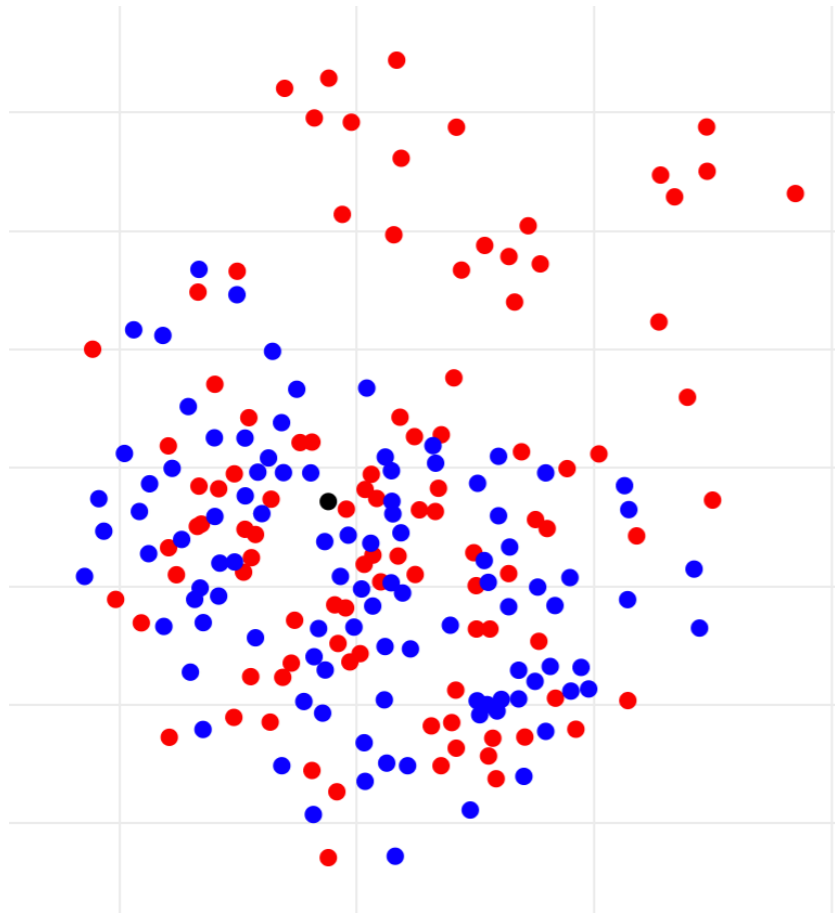


Visualizations of
Tree Space

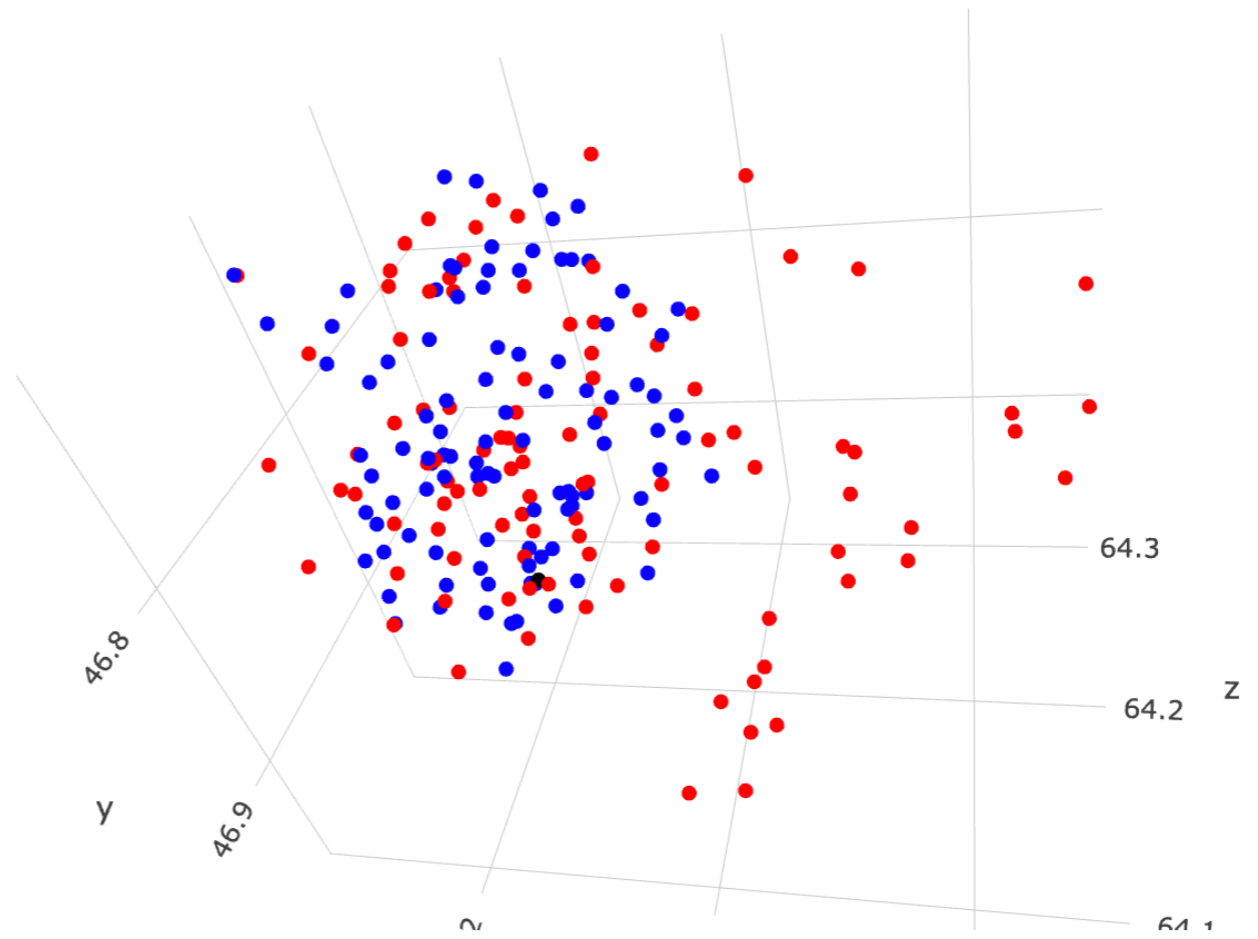
```
run1Trees = readTrees("run1.trees")  
run2Trees = readTrees("run2.trees")  
maxdiff([run1Trees,run2Trees])
```

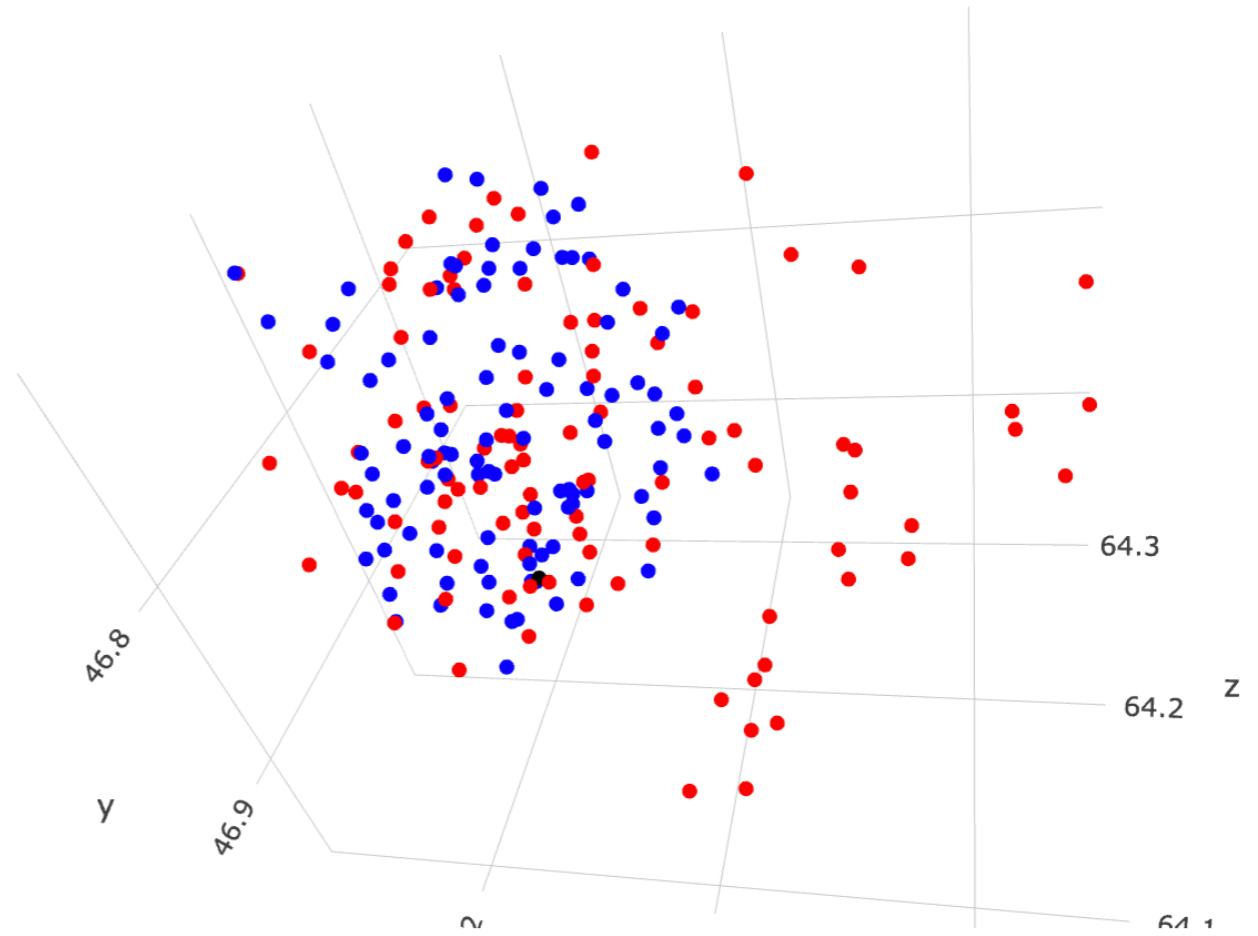
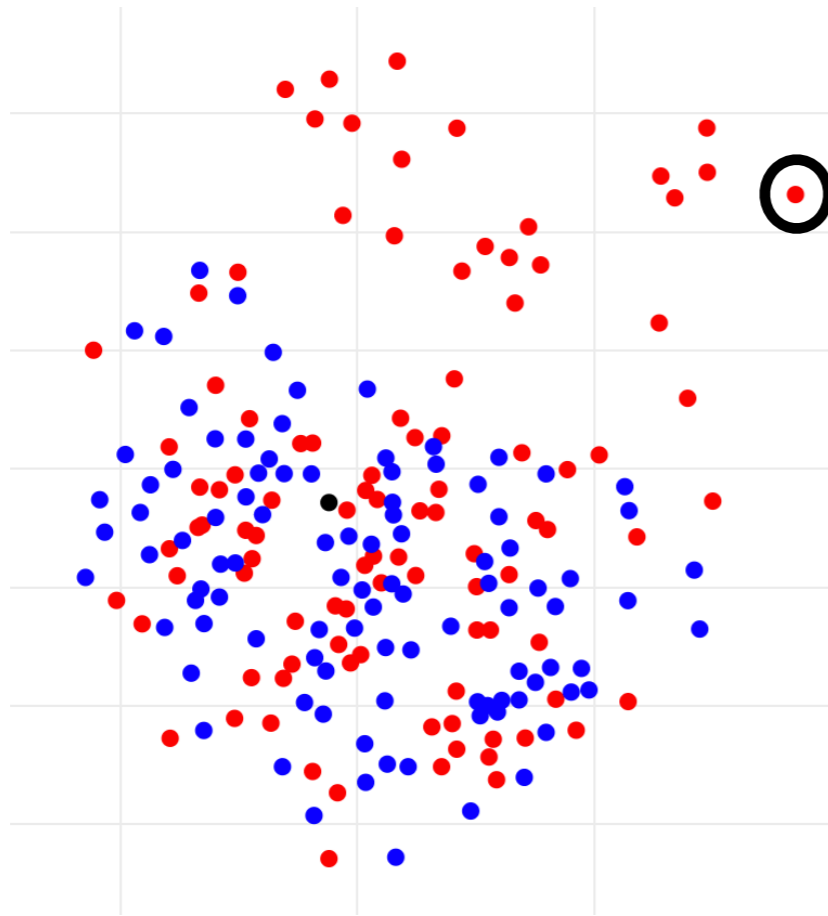
Finds the biggest difference in estimated marginal posterior probabilities for all bipartitions across different replicates.

2D

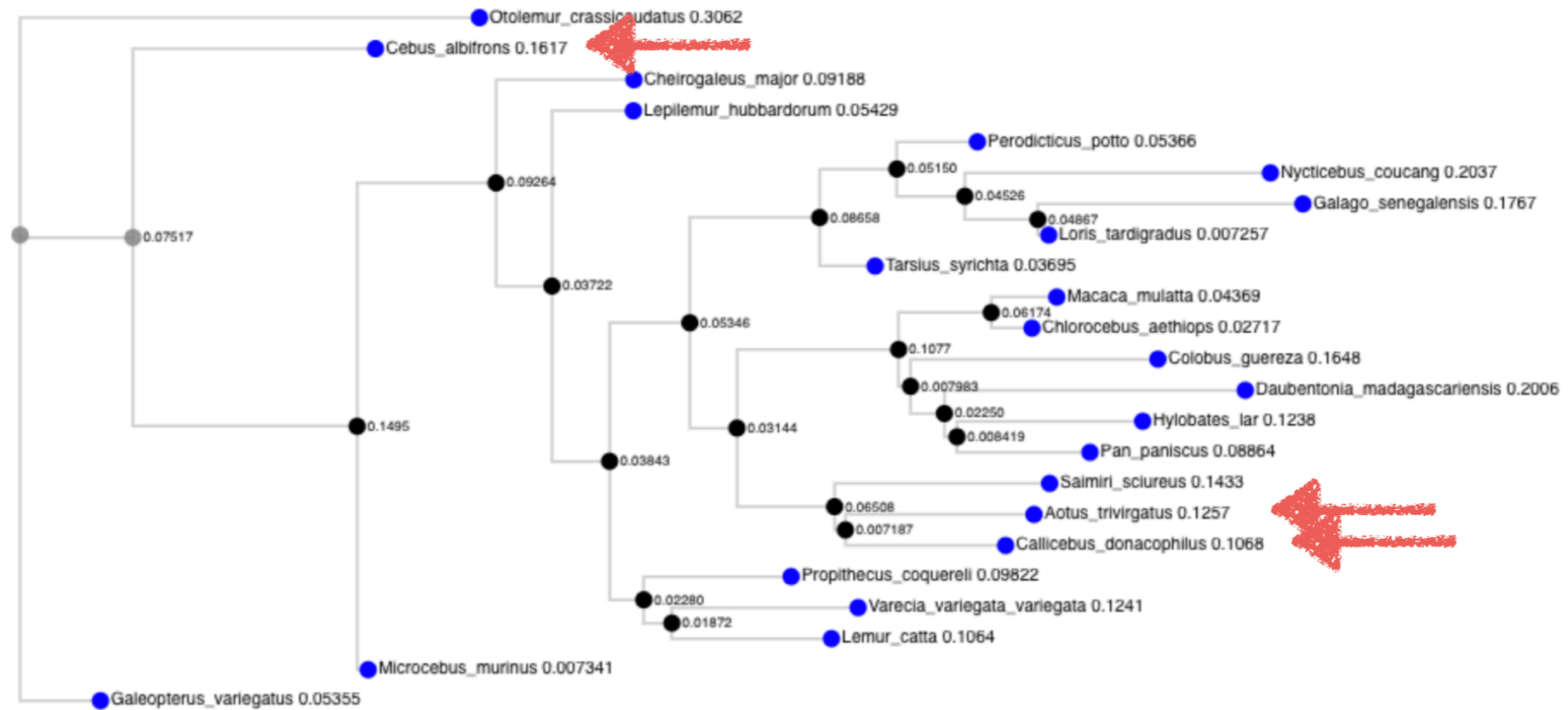


3D





Tree 10



RevBayes Tutorial Setup

After logging on to your VM...

```
cd ~  
  
cp -r moledata/revbayes/ ./  
  
cd revbayes/genetree/  
  
mv ./scripts/*.Rev .
```

RevBayes CTMC Tutorial

<https://revbayes.github.io/tutorials/ctmc/>

I strongly encourage you to try building at least one Rev script yourself for this tutorial, rather than simply running those that are already available.

Also note that in the scripts that are already written, the default number of replicate runs is 4. This is great for assessing convergence, but will cause analyses to run fairly slowly. You can reduce this to 2 to make sure that runs finish in a reasonable amount of time.

You'll also want to change the `printgen` option of `mnModel` and `mnFile` to 10 (rather than 1). Otherwise, output files get huge!

RevBayes CTMC Tutorial

<https://revbayes.github.io/tutorials/ctmc/>

Look at Table 1 in the tutorial above.

Using the model descriptions in the table, use the same principles we've been practicing to set up each of these models in RevBayes.

Fill out Table 2 for as many models as you can.

```
rb mcmc_JC.Rev
```

End